CHAPTER

10

Natural (or Free) Convection

10.1 Introduction

In the previous chapter, we studied heat transfer by forced convection, wherein fluid movement was caused by an external agency such as a pump or fan. In this chapter, we shall study about heat transfer in 'Natural or free convection'; here, fluid movement is caused because of density differences in the fluid due to temperature differences, under the influence of gravity. Density differences cause a 'buoyancy force' which in turn, causes the fluid circulation by 'convection currents'. Buoyancy force is the upward force exerted by a fluid on a completely or partially immersed body and is equal to the *weight of the fluid* displaced by the body. Obviously, fluid velocity in natural convection is low as compared to that in forced convection, and as a result, the heat transfer coefficient is also lower in the case of natural convection. Still, natural convection is one of the important modes of heat transfer used in practice since there are no moving parts and as a result, there is an increased reliability. Natural convection heat transfer is extensively used in the following areas of engineering:

(i) cooling of transformers, transmission lines and rectifiers
(ii) heating of houses by steam or electrical radiators
(iii) heat loss from steam pipe lines in power plants and heat gain in refrigerant pipe lines in air-conditioning applications
(iv) cooling of reactor core in nuclear power plants
(v) cooling of electronic devices (chips, transistors, etc.) by finned heat sinks.

10.2 Physical Mechanism of Natural Convection

Consider the familiar example of a heated, vertical plate kept hanging in quiescent air. Let the temperature of the heated surface be T_s and that of the surrounding air, T_a. A layer of air in the immediate vicinity of the plate will get heated by conduction; density of this heated air layer decreases (since the total pressure of surroundings is constant and $p = \rho R_{air} T$ for an ideal gas). As a result, the heated layer rises up and the cold air from the surroundings moves in to take its place. This layer, in turn, gets heated up, moves up and is again replaced by cooler air etc. Thus, convection currents are set up causing the heat to be carried away from the hot surface. This situation is shown in Fig. 10.1 (a).

During the temperature induced flow, a boundary layer is set up along the length of the plate as shown. With the x-axis taken along the vertical length of plate, and the y-axis perpendicular to it, the velocity and temperature profiles are shown in the Fig. 10.1. As far as the velocity profile is concerned, at the plate surface, the fluid velocity is zero due to 'no slip' condition; then, the velocity increases to a maximum value and then, drops to zero at the outer edge of the boundary layer since the surrounding air is assumed to be quiescent. Note the difference in this velocity profile as compared to that in the case of forced convection. The boundary layer is laminar for some distance along the length, and then depending on the fluid properties and the driving temperature difference between the wall and the ambient, the boundary layer becomes turbulent.

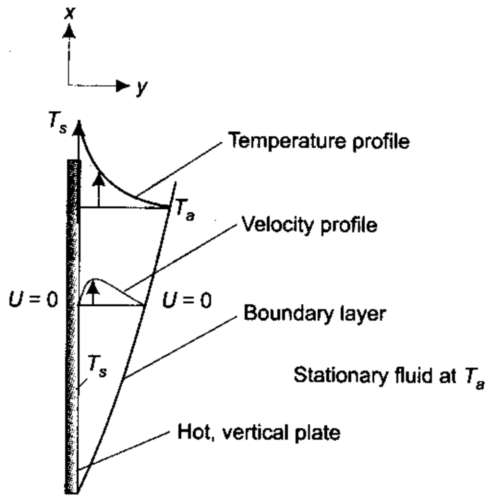

FIGURE 10.1(a) Velocity and temperature profiles in natural convection

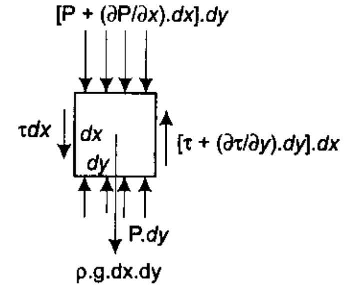

FIGURE 10.1(b) Differential control volume with forces for natural convection

Analytical solution of natural convection heat transfer is a little more complicated since velocity field is coupled to the temperature field because the flow is induced by temperature differences. Temperature field is also coupled to the velocity field, i.e. we say that the velocity and temperature fields are 'mutually coupled'. Therefore, to get a solution, momentum and energy equations for the boundary layer have to be solved simultaneously. Solutions by the exact and approximate integral methods have been obtained for the simple cases, but the predicted surface heat transfer coefficients are smaller than the experimentally measured values, because the analysis do not take into account rate-increasing disturbances (remember: velocities are quite small in free convection) present in actual equipments. Therefore, in handling natural convection problems, we rely mostly on empirical relations derived as a result of large experimental work.

10.3 Dimensional Analysis of Natural Convection—Grashoff Number

Natural convection heat transfer is a good candidate for dimensional analysis since we can reliably list the parameters on which this phenomenon depends and the theoretical analysis is rather difficult and we have to depend mostly on experimental work.

As we stated earlier, in natural convection, flow is induced by the density differences caused as a result of temperature differences. In the gravitational field, the density differences induce a buoyancy force given by:

$$F_b = \rho_{\text{fluid}} \cdot g \cdot V_{\text{body}} \qquad \text{...(10.1)}$$

where ρ_{fluid} = density of fluid

g = acceleration due to gravity

V_{body} = volume of portion of body immersed in fluid

In the absence of other body forces (such as centrifugal, electromagnetic etc.),

Net vertical force acting on the body = weight of body – buoyancy force

i.e. $$F_{\text{net}} = W - F_b$$

i.e. $$F_{\text{net}} = \rho_{\text{body}} \cdot g \cdot V_{\text{body}} - \rho_{\text{fluid}} \cdot g \cdot V_{\text{body}}$$

i.e. $$F_{\text{net}} = (\rho_{\text{body}} - \rho_{\text{fluid}}) \cdot g \cdot V_{\text{body}} \qquad \text{...(10.2)}$$

Now, the density differences can be related to the temperature differences by the temperature coefficient of volumetric expansion, β, which is defined as:

$$\beta = (1/v).(\partial v/\partial T)_p = -(1/\rho).(\partial \rho/\partial T)_p \qquad \text{....(1/K)} \quad \text{...(10.3)}$$

i.e. $$\beta = -(1/\rho).(\Delta\rho/\Delta T) \text{ at constant } P$$

i.e. $$\Delta\rho = -\rho.\beta.\Delta T \text{ at constant } P$$

For ideal gas, $p = \rho.R.T$ and, $\beta = 1/T$, where T is expressed in Kelvin.

TABLE 10.1 Parameters for Natural convection heat transfer

Sl. No.	Parameter	Symbol (Unit)	Primary dimension
1	*Significant length*	L_c (m)	L
2	Fluid density	ρ (kg/m^3)	ML^{-3}
3	Fluid viscosity	μ (kg/(m.s))	$ML^{-1}t^{-1}$
4	Temperature difference	ΔT (deg.C)	T
5	Coefficient of volume expansion	β (1/K)	T^{-1}
6	Acceleration due to gravity	g (m/s^2)	Lt^{-2}
7	Th. conductivity of fluid	k (W/(m.K))	$MLt^{-3}T^{-1}$
8	Heat transfer coefficient	h (W/(m^2.K))	$Mt^{-3}T^{-1}$
9	Specific heat of fluid	C_p (J/(kg.K))	$L^2t^{-2}T^{-1}$

With this background, now let us list out the parameters (and their primary dimensions) on which the phenomenon of Natural convection heat transfer depends, as shown in Table 10.1.

We see that there are 9 variables listed. Of these, the product $\beta.g.\Delta T$ represents buoyancy forces and is considered as a single variable. Thus, we can say that there are 7 variables affecting the phenomenon and there are 4 primary dimensions, viz. (M, L, T and t).

Then, from Buckingham theorem, Number of independent dimensionless groups that can be formed is equal to 7 – 4 = 3.

Choosing L_c, ρ, μ and k as the core group, we write:

$$\pi_1 = L_c^a \cdot \rho^b \cdot \mu^c \cdot k^d \cdot (g \cdot \beta \cdot \Delta T) \quad \text{...(a)}$$

$$\pi_2 = L_c^p \cdot \rho^q \cdot \mu^r \cdot k^s \cdot C_p \quad \text{...(b)}$$

$$\pi_3 = L_c^w \cdot \rho^x \cdot \mu^y \cdot k^z \cdot h \quad \text{...(c)}$$

(i) Considering Eq. a:

$$\pi_1 = M^0 \cdot L^0 \cdot T^0 \cdot t^0 = L^a \cdot (M \cdot L^{-3})^b \cdot (M \cdot L^{-1} \cdot t^{-1})^c \cdot (M \cdot L \cdot t^{-3} \cdot T^{-1})^d \cdot (L \cdot t^{-2})$$

Equating the coefficients of M, L, T and t on either side of above equation., we get:

M: $0 = b + c + d$

L: $0 = a - 3b - c + d + 1$

T: $0 = -d$

t: $0 = -c - 3d - 2$

Solving the above set of equations simultaneously, we get:

$d = 0, c = -2, b = 2, a = 3$

Therefore, π_1, the first dimensionless group is:

$$\pi_1 = \frac{\rho^2 \cdot (g \cdot \beta \cdot \Delta T) \cdot L_c^3}{\mu^2} = \frac{(g \cdot \beta \cdot \Delta T) \cdot L_c^3}{\nu^2} = Gr$$

where Gr = Grashoff number

(ii) Considering Eq. b:

$$\pi_2 = M^0 \cdot L^0 \cdot T^0 \cdot t^0 = L^p \cdot (M \cdot L^{-3})^q \cdot (M \cdot L^{-1} \cdot t^{-1})^r \cdot (M \cdot L \cdot t^{-3} \cdot T^{-1})^s \cdot (L^2 \cdot t^{-2}\, T^{-1})$$

Equating the coefficients of M, L, T and t on either side of above equation., we get:

M: $0 = q + r + s$

L: $0 = p - 3q - r + s + 2$

T: $0 = -s - 1$

t: $0 = -r - 3s - 2$

Solving the above set of equations simultaneously, we get:

$s = -1, r = 1, q = 0, p = 0$

Therefore, π_2, the second dimensionless group is:

$$\pi_2 = \frac{\mu \cdot C_p}{k} = Pr$$

where, Pr = Prandtl number

(iii) Considering Eq. c:

$$\pi_3 = M^0 \cdot L^0 \cdot T^0 \cdot t^0 = L^w \cdot (M \cdot L^{-3})^x \cdot (M \cdot L^{-1} \cdot t^{-1})^y \cdot (M \cdot L \cdot t^{-3} \cdot T^{-1})^z \cdot (M \cdot t^{-3} \cdot T^{-1})$$

Equating the coefficients of M, L, T and t on either side of above equation, we get:

M: $0 = x + y + z + 1$

L: $0 = w - 3x - y + z$

T: $0 = -z - 1$

t: $0 = -y - 3z - 3$

Solving the above set of equations simultaneously, we get:

$z = -1, y = 0, x = 0, w = 1$

Therefore, π_3, the third dimensionless group is:

$$\pi_3 = \frac{h \cdot L}{k} = Nu$$

where, Nu = Nusselt number

Thus, for natural convection, we have:

$$Nu = f(Gr, Pr)$$

Of course, the exact form of equation with associated constants must be determined from experiments.

As we shall see later, in most of the empirical relations, product of Gr and Pr is taken together and the relations are presented in the form:

$$Nu = C.Ra^m$$

where

$Ra = (Gr.Pr)$ = Rayleigh number and 'C' and 'm' are constants determined from experiments.

By determining Nu, we determine h, the heat transfer coefficient in natural convection.

Then, the heat transfer rate for natural convection is given by Newton's law of cooling, i.e.

$$Q_{conv} = h.A.(T_s - T_a), \text{ W}$$

Thus, from dimensional analysis, we have established that, in natural convection heat transfer problems, dimensional groups of significance are: Grashoff number (Gr), Prandtl number (Pr) and Nusselt number (Nu). Out of these, Gr plays the same role in natural convection as that of Reynolds number in forced convection.

Physical significance of Grashoff number is that it represents the ratio of buoyancy force to the viscous force acting on the fluid, i.e.

$$G_r = \frac{\text{Buoyancy forces}}{\text{Viscous forces}} = \frac{g \cdot \Delta\rho V}{\rho \cdot \nu^2} = \frac{g \cdot \beta \cdot \Delta T \cdot V}{\nu^2}$$

since $\Delta\rho = \rho.\beta.\Delta T$.

So, we can write:

$$G_r = \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L_c^3}{\nu^2} \qquad ...(10.4)$$

where,

g = acceleration due to gravity, m/s^2

β = coefficient of volume expansion, $1/K$ ($\beta = 1/T$ for ideal gases only, T in Kelvin)

T_s = temperature of the surface, deg. C

T_a = temperature of the fluid at sufficient distance from the surface, deg. C

L_c = characteristic length of the geometry, m, and

ν = kinematic viscosity of fluid, m^2/s

Product of Grashoff number and Prandtl number, i.e. Rayleigh number, $Ra = Gr.Pr$ is the criterion to determine if the flow is laminar or turbulent, in natural convection. For example, in the case of heat transfer by natural convection for vertical plates, for $Ra >$ about 10^9, the flow is turbulent and for $Ra < 10^9$, the flow is laminar.

10.4 Governing Equations and Solution by Integral Method

As stated earlier, in solving natural convection heat transfer problems, we rely more on empirical relations than on analytical relations. This is due to the fact that analytical relations are rather difficult to obtain since the momentum and energy equations are 'mutually' coupled; also, analytical relations generally give lower values of heat transfer coefficients as compared to empirical relations. Also, because of the very low velocities involved in natural convection, it becomes difficult to take into account all factors in the analytical methods and the insertion of measuring probes itself introduces disturbances in the flow fields.

So, we shall indicate the development of governing equations for the simple case of heat transfer by free convection from a heated, vertical plate at a surface temperature T_s and the surrounding ambient at a temperature T_a, and very briefly give the outline of the solution by the approximate Integral method and give only the final results.

In general, solution of the problem involves the simultaneous solution of equations of continuity, momentum and energy.

Equation of continuity:

Considering an elemental volume in the two-dimensional boundary layer, with constant properties, the continuity equation remains the same as derived for forced convection, i.e. Eq. 9.15. We rewrite it here as:

$$(\partial u/\partial x) + (\partial v/\partial y) = 0 \qquad \text{...(10.5)}$$

Equation of momentum:

This is derived by applying Newton's second law to the differential control volume. Fig. 10.1 (b) shows the various forces acting on the control volume. Net force acting in the x-direction ($= \Sigma F_x$) must be equal to the rate of change of momentum in that direction.

i.e.
$$\Sigma F_x = \rho.\{u.(\partial u/\partial x) + v.(\partial u/\partial y)\}.dx.dy \qquad \text{...(Eq. A)}$$

See Eq. a under section 9.7.2

As far as ΣF_x is concerned, compared to the case of forced convection, now there an additional force due to gravity, acting in the downward direction i.e. opposed to the positive X-direction; so, we get

$$\Sigma F_x = -(\partial p/\partial x).dx.dy - \rho.g.dx.dy + \mu(\partial^2 u/\partial y^2).dx.dy \qquad \text{...(Eq. B)}$$

See Eq. *b* under section 9.7.2

Equating Eqs. A and B, we get:

$$\rho.\{u.(\partial u/\partial x) + v.(\partial u/\partial y)\} = -(\partial p/\partial x) - \rho.g + \mu(\partial^2 u/\partial y^2) \qquad \text{...(10.6)}$$

Since the pressure gradient in X-direction is due to change in elevation of plate, we write:

$$(\partial p/\partial x) = -\rho_a.g$$

Therefore, Eq. 10.6 becomes:

$$\rho.\{u.(\partial u/\partial x) + v.(\partial u/\partial y)\} = g.(\rho_a - \rho) + \mu(\partial^2 u/\partial y^2) \qquad \text{...(10.7)}$$

Now, the density difference $(\rho_a - \rho)$ may be related to the temperature difference as follows:

$$\beta = (1/V).(\partial V/\partial T)_p = (1/V).\{(V - V_a)/(T - T_a)\} = (\rho_a - \rho)\ /\{\rho.(T - T_a)\}$$

So, we get:

$$\rho.\{u.(\partial u/\partial x) + v.(\partial u/\partial y)\} = g.\rho.\beta(T - T_a) + \mu(\partial^2 u/\partial y^2) \qquad \text{...(10.8)}$$

Eq. 10.8 is the momentum equation for natural convection boundary layer; note that its solution requires a knowledge of temperature distribution.

In general, volume coefficient of expansion, β for fluids has to be obtained from data tables; however, for ideal gases, $\beta = 1/T$, where T is the absolute temperature in Kelvin.

Equation of energy:

Again, equation of energy remains the same as derived earlier for forced convection, equation 9.18. We rewrite it here as:

$$u.(\partial T/\partial x) + v.(\partial T/\partial y) = \alpha.(\partial^2 T/\partial y^2). \qquad \text{...(10.9)}$$

While solving this problem by the approximate integral method, we make an assumption that the fluid is incompressible except for the effect of variable density in the buoyancy force, since fluid motion is induced by this variation. This is known as Boussinesq approximation. And, the flow is with laminar boundary layer, steady, two-dimensional and with constant fluid properties. Further, since it is the temperature difference that induces the flow in natural convection, both the hydrodynamic and thermal boundary layers are assumed to be identical, i.e. thicknesses of both the boundary layers are assumed to be equal, or $\delta = \delta_t$.

Integrating Eq. 10.8 over the boundary layer thickness, we get the **integral momentum equation:**

$$\frac{d}{dx}\left(\int_0^{\delta} \rho \cdot u^2\, dy\right) = \tau_s + \int_0^{\delta} \rho \cdot g \cdot \beta \cdot (T - T_a)dy$$

i.e. $$\frac{d}{dx}\left(\int_0^{\delta} \rho \cdot u^2\, dy\right) = -\mu \cdot \left(\frac{du}{dy}\right)_{y=0} + \int_0^{\delta} \rho \cdot g \cdot \beta \cdot (T - T_a)dy \qquad ...(10.10)$$

To solve this, we have to assume the velocity and temperature distributions which satisfy the boundary conditions, just as we did in the case of forced convection.

For temperature distribution, the boundary conditions are:

$$T = T_s \text{ at } y = 0$$

$$T = T_a \text{ at } y = \delta$$

and, $$(\partial T/\partial y) = 0 \text{ at } y = \delta$$

And the temperature distribution which satisfies these conditions is:

$$\frac{T - T_a}{T_s - T_a} = \left(1 - \frac{y}{\delta}\right)^2 \qquad ...(10.11)$$

Boundary conditions for the velocity profile are:

$$u = 0 \text{ at } y = 0$$

$$u = 0 \text{ at } y = \delta$$

and, $$(\partial u/\partial y) = 0 \text{ at } y = \delta$$

An additional condition from Eq. 10.8 is:

$(\partial^2 u/\partial y^2) = -g.\beta.(T_s - T_a)/\nu$ at $y = 0$ since both u and v are zero at the surface.

And the velocity profile which satisfies these conditions is:

$$\frac{u}{u_x} = \frac{y}{\delta} \cdot \left(1 - \frac{y}{\delta}\right)^2 \qquad ...(10.12)$$

Here, u_x is a fictitious reference velocity, an arbitrary function of x, since there is no free stream velocity in natural convection.

Maximum velocity and its position is determined by differentiating Eq. 10.12 w.r.t y and equating to zero. The result is:

$$u_{max} = \frac{4}{27} \cdot u_x \quad \text{at } y = \delta/3 \qquad ...(10.12a)$$

And, the **mean velocity** at a section is obtained by integrating the velocity function over the boundary layer thickness:

$$u_m = \frac{1}{\delta} \cdot \int_0^{\delta} u dy = \frac{1}{\delta} \cdot \int_0^{\delta} u_x \cdot \left(\frac{y}{\delta}\right) \cdot \left(1 - \frac{y}{\delta}\right)^2 dy$$

i.e. $$u_m = \frac{1}{12} \cdot u_x = \frac{27}{48} \cdot u_{max} \qquad ...(10.12b)$$

Inserting Eqs. 10.11 and 10.12 in Eq. 10.10 and performing the mathematical operations, one gets:

$$\frac{1}{105} \cdot \frac{d(u_x^2 \cdot \delta)}{dx} = \frac{1}{3} \cdot g \cdot \beta \cdot (T_s - T_a) \cdot \delta = \frac{\nu \cdot u_x}{\delta} \qquad ...(10.13)$$

Similarly, integrating Eq. 10.9, we get the **integral form of energy equation** as follows:

$$\frac{d}{dx}\left[\int_0^{\delta} u \cdot (T - T_a)dy\right] = -\alpha \cdot \left(\frac{dT}{dy}\right)_{y=0} \qquad ...(10.14)$$

Substituting the assumed velocity and temperature distributions in Eq. 10.14, final result is:

$$\frac{1}{30} \cdot (T_s - T_a) \cdot \frac{d(u_x \cdot \delta)}{dx} = 2 \cdot \alpha \cdot \frac{(T_s - T_a)}{\delta} \qquad ...(10.15)$$

Assuming exponential functional variations for u_x and δ, i.e. $u_x = C_1 . x^{1/2}$ and $\delta = C_2 . x^{1/4}$, we get the final result for velocity function and boundary layer thickness in laminar flow as:

$$u_x = 5.17 \cdot \nu \cdot (Pr + 0.952)^{-0.5} \cdot \left[\frac{\beta \cdot g \cdot (T_s - T_a)}{\nu^2}\right]^{0.5} \cdot x^{0.5} \quad ...(10.16a)$$

and,

$$\frac{\delta}{x} = \frac{3.93 \cdot (0.952 + Pr)^{0.25}}{Gr_x^{0.25} \cdot Pr^{0.5}} \quad ...(10.16b)$$

Eq. 10.16b gives the variation of δ along the height x of the plate. Gr_x is the Grashoff number.

Mass flow rate through the boundary:

Mass flow rate through a section for unit width of plate is given by:

$$m = u_m \cdot (\delta \cdot 1) \cdot \rho = \frac{u_x}{12} \cdot \delta \cdot \rho = \frac{\rho}{12} \cdot (\delta \cdot u_x) \quad ...(10.17)$$

u_x and δ are obtained from Eq. 10.16 a & b.

Mass flow between two sections at x_1 and x_2 can be determined by the difference in values of m (as calculated from Eq. 10.17) between these two sections.

Total mass flow through the boundary is obtained by putting $x_1 = 0$ and $x_2 = L$. We get:

$$m_{\text{total}} = 1.7 \cdot \rho \cdot \nu \cdot \left[\frac{Gr_L}{Pr^2 \cdot (Pr + 0.952)}\right]^{0.25} \quad ...(10.18)$$

Heat transfer coefficient is determined from

$$q_s = -k \cdot A \cdot \left(\frac{dT}{dy}\right)_s = h \cdot A \cdot (T_s - T_a)$$

Using the temperature distribution given by Eq. 10.11, we get:

$$h = \frac{2 \cdot k}{\delta}$$

i.e.

$$\frac{h \cdot x}{k} = Nu_x = 2 \cdot \frac{x}{\delta}$$

And the dimensionless heat transfer coefficient (i.e. Nusselts number) is:

$$Nu_x = \frac{0.508k2\,Pr^{0.5} \cdot Gr_x^{0.25}}{(0.952 + Pr)^{0.25}} \quad \textit{(using eqn. 10.16(b))}...(10.19)$$

Average heat transfer coefficient for the vertical plate is obtained by integrating over the height L:

$$h_{\text{avg}} = \frac{1}{L} \cdot \int_0^L h_x \, dx$$

i.e.

$$h_{\text{avg}} = \frac{4}{3} \cdot h_L \quad ...(10.20a)$$

and,

$$Nu_{\text{avg}} = \frac{4}{3} \cdot Nu_L = \frac{0.667 \cdot Pr^{0.5} \cdot Gr_L^{0.25}}{(0.952 + Pr)^{0.25}} \quad ...(10.20b)$$

Note that for forced convection over a flat plate, we had $Nu_{\text{avg}} = 2.\ Nu_L$

Above equations are valid for laminar boundary layer flow only

For turbulent boundary layer flow, $(Gr \cdot Pr > 10^9)$, by following the integral method, we get:

$$\frac{\delta_{\text{turb}}}{x} = \frac{0.565 \cdot \left(1 + 0494 \cdot Pr^{\frac{2}{3}}\right)}{Gr^{0.1} \cdot Pr^{\frac{8}{15}}} \quad (Gr \cdot Pr = Ra = \textit{Rayleigh number})...(10.21)$$

and,

$$Nu_{avg} = \frac{h_{avg} \cdot L}{k} = 0.0246 \left[\frac{Pr^{1.17} \cdot Gr_L}{1 + 0.495 \cdot Pr^{\frac{2}{3}}} \right]^{0.4} \qquad \textit{(for turbulent flow...(10.22))}$$

In the above equation physical properties of fluid are taken at the average (film) temperature, i.e.

$$T_f = \frac{T_s + T_a}{2}$$

An outline of the analytical procedure involved for the simple case of heat transfer by convection from a heated vertical plate is presented above just to illustrate the fact that even for simple cases, analytical procedures are rather involved. It is stated again, that this is due to the mutual coupling of momentum and energy equations.

10.5 Empirical Relations For Natural Convection Over Surfaces and Enclosures

Free convection patterns from a few common geometries are shown in Fig. 10.2.

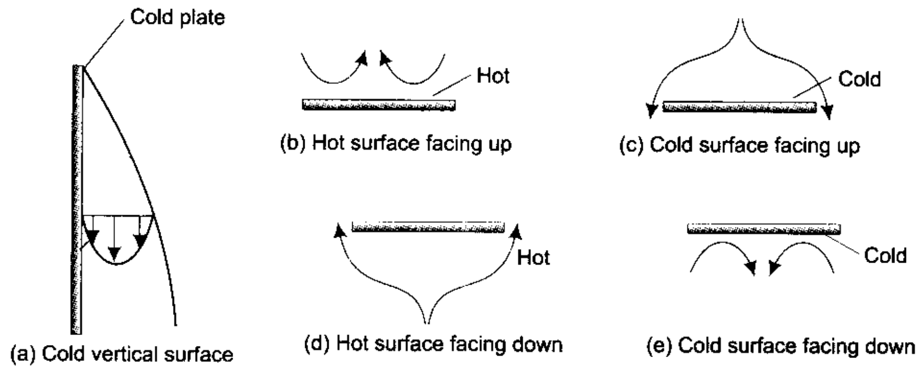

FIGURE 10.2 Free convection flow patterns

We shall present below empirical relations for natural convection from several types of surfaces and enclosures of practical importance. While using the empirical relations, it is important to remember the conditions under which these relations are valid. Observe that most of the relations are presented in the form: $Nu = C.Ra^m$, where C and m are constants deduced from experiments. Nu is the Nusselt number ($= h.L_c/k$), Ra is the Rayleigh number ($= Gr.Pr$); characteristic dimension L_c for vertical plates and cylinders is generally the plate (or cylinder) height L or diameter D for a horizontal cylinder.

10.5.1 Vertical Plate at Constant Temperature T_s

Vertical plate is an important geometry since heat transfer from the walls of a furnace can be calculated by the relations applicable to a vertical plate.

McAdams has suggested the following relations for fluids whose Prandtl number is close to unity, i.e. for air and other gases, generally:

$$Nu = 0.59 \cdot Ra^{\frac{1}{4}} \qquad ...10^4 < Ra < 10^9...(10.23)$$

and

$$Nu = 0.13 \cdot Ra^{\frac{1}{3}} \qquad ...10^9 < Ra < 10^{12}...(10.24)$$

Eq. 10.23 is for laminar, boundary layer type, natural convection flow, while Eq. 10.24 is for turbulent, boundary layer type, natural convection flows. Fluid properties are evaluated at film temperature T_f, already defined.

Churchill and Chu present following relations for the entire range of *Ra* and also valid for all Prandtl numbers from 0 to ∞.

For $0 < Ra < 10^9$, $0 < Pr < \infty$:

$$Nu = 0.68 + \frac{0.670 \cdot Ra^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} \qquad (0 < Ra < 10^9 \ldots (10.25))$$

For $Ra > 10^9$, $0.6 < Pr < \infty$:

$$Nu = \frac{0.15 \cdot Ra^{\frac{1}{3}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{27}}} \qquad (Ra > 10^9 \ldots (10.26))$$

For $Ra > 10^9$, $0 < Pr < 0.6$:

$$Nu = \left[0.825 + \frac{0.387 \cdot Ra^{\frac{1}{6}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2 \qquad (Ra > 10^9 \ldots (10.27))$$

Eq. 10.25 is for fluids whose *Pr* is not too close to unity (or, to that of air). Eq. 10.26 is for high Prandtl no. fluids, and Eq. 10.27 is for low *Pr* fluids i.e. for liquid metals.

In the above equations characteristic dimension for *Nu* and *Ra* is the height *L* of the plate; fluid properties are evaluated at the film temperature T_f.

For **inclined plates** (inclined at an angle θ to the vertical), vertical plate relations can be used by replacing *g* by $g.\cos(\theta)$ for $Ra < 10^9$. Inclined length *L* is the characteristic dimension.

10.5.2 Vertical Cylinders At Constant Temperature T_s

A vertical cylinder can be treated as a vertical plate and the relations given above can be applied if the following criterion is satisfied:

$$\frac{D}{L} \geq \frac{34}{Ra^{\frac{1}{4}}} \qquad \ldots(10.28)$$

Height *L* of the cylinder is the characteristic dimension.

10.5.3 Vertical Plate With Constant Heat Flux

Equations of Churchill and Chu, 10.25 and 10.26 are valid, with the following modifications: (a) temperature of the constant flux plate is considered at a point mid-way between top and bottom (b) constant 0.492 should be changed to 0.437.

Alternative relations are given below for vertical and inclined plates for natural convection in water and air. Here, a modified Grashoff number, *Gr'* is defined:

$$Gr' = Gr \cdot Nu_x = \frac{g \cdot \beta \cdot q_s \cdot x^4}{k \cdot \nu^2} \qquad \ldots(10.29)$$

where q_s is the wall heat flux in W/m^2. Then the following two relations are recommended for local heat transfer coefficients in laminar and turbulent ranges respectively:

$$Nu_x = 0.60 \cdot (Gr' \cdot Pr)^{0.2} \qquad (10^5 < Gr_x' < 10^{11} ...(10.30))$$

and,

$$Nu_x = 0.17 \cdot (Gr' \cdot Pr)^{0.25} \qquad (Gr_x' > 10^{11} ...(10.31))$$

And the average heat transfer coefficient in the laminar region is obtained by integration over the entire height L of the plate as:

$$h = \frac{5}{4} \cdot h_L \qquad \textit{(for laminar...(10.32))}$$

and, for turbulent region, h_x is independent of x:

$$h = h_L \qquad \textit{(for turbulent...(10.33))}$$

Example 10.1. A hot plate 30 cm high and 1.2 *m* wide at 140°C is exposed to ambient air at 20°C. Using the approximate solution, calculate the following:

(i) Maximum velocity at 12 cm from the leading edge of the plate (ii) boundary layer thickness at 12 cm from the leading edge of plate (iii) local heat transfer coefficient at 12 cm from the leading edge of the plate (iv) average heat transfer coefficient over the surface of the plate (v) total mass flow through the boundary (vi) total heat loss from the plate, and (vii) temperature rise of air

Solution.

Data:

$L := 0.3$ m $\quad W := 1.2$ m $\quad Ts := 140°C \quad Ta := 20°C \quad x := 0.12$ m $\quad g := 9.81$ m/s^2

We need properties of air at film temperature $T_f = (140 + 20)/2$

$$T_f := 80°C$$

Properties of air at 80°C:

$\rho := 1.00$ kg/m^3 $\quad \nu := 21.09 \times 10^{-6}$ m^2/s $\quad Pr := 0.692 \quad k := 0.03047$ W/(mK) $\quad Cp := 1009$ J/(kgK)

$$\beta := \frac{1}{(T_f + 273)} \qquad \textit{(coefficient of volume expansion...}\textbf{\textit{Note that temperature must be in Kelvin}}\textit{)}$$

i.e. $\quad \beta = 2.833 \times 10^{-3}$ 1/k

Grashoff number:

At $x = 0.12$ m: $\quad Gr_x := \dfrac{g \cdot \beta \cdot (T_s - T_a) \cdot x^3}{\nu^2}$

i.e. $\quad Gr_x = 1.296 \times 10^7$

At $L = 0.3$ m: $\quad Gr_L := \dfrac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2}$

i.e. $\quad Gr_L = 2.024 \times 10^8$

(i) Maximum velocity

To calculate this, first we need the velocity function u_x. We have from Eq. 10.16a:

$$u_x := 5.17 \cdot \nu \cdot (Pr + 0.952)^{-0.5} \cdot \left[\frac{\beta \cdot g \cdot (T_s - T_a)}{\nu^2}\right]^{0.5} \cdot x^{0.5} \qquad ...(10.16a)$$

i.e. $\quad u_x = 2.551$ m/s $\qquad$ *(velocity function)*

Therefore, maximum velocity is given by:

$$u_{max} := \frac{4}{27} \cdot u_x \qquad ...(10.12a)$$

i.e. $\quad u_{max} = 0.378$ **m/s**

(ii) Thickness of boundary layer:

We have:

$$\frac{\delta}{x} = \frac{3.93 \cdot (0.952 + Pr)^{0.25}}{Gr_x^{0.25} \cdot Pr^{0.5}} \qquad ...(10.16b)$$

i.e.

$$\delta := \frac{3.93 \cdot (0.952 + Pr)^{0.25}}{Gr_x^{0.25} \cdot Pr^{0.5}} \cdot x$$

i.e. $\quad \delta = 0.0107$ *m* = **10.7 mm.**

(iii) Heat transfer coefficient

We have:

$$Nu_x := \frac{0.508 \cdot Pr^{0.5} \cdot Gr_x^{0.25}}{(0.952 + Pr)^{0.25}} \quad ...(10.19)$$

i.e. $Nu_x = 22.39$ (*Check* $Nu_x = 2.x/\delta = 22.43$*...checks*)

Therefore, $h_x := \frac{Nu_x \cdot k}{x}$

i.e. $h_x = 5.685$ **W/(m²K)** ***(heat transfer coefficient)***

Heat transfer coefficient at $x = L$:

$$h_L = \frac{Nu_L \cdot k}{L}$$

i.e. $$h_L := \frac{0.508 \cdot Pr^{0.5} \cdot Gr_l^{0.25}}{(0.952 + Pr)^{0.25}} \cdot \frac{k}{L}$$

i.e. $h_L = 4.521$ W/(m²K) (*heat transfer coefficient at* $x = L$)

(iv) Average value of heat transfer coefficient:

$$h_{avg} := \frac{4}{3} \cdot h_L \quad ...(10.20a)$$

i.e. $h_{avg} = 6.028$ **W/(m²K)** ***(average heat transfer coefficient)***

(v) Total mass flow rate through boundary layer:

We have, from Eq. 10.18

$$m_{total} := 1.7 \cdot \rho \cdot \nu \cdot \left[\frac{Gr_L}{Pr^2 \cdot (Pr + 0.952)}\right]^{0.25} \quad ...(10.18)$$

i.e. $m_{total} = 4.54 \times 10^{-3}$ **kg/s.**

(vi) Heat transfer from the plate:

$Q = h_{avg} \cdot A_s \cdot (T_s - T_a)$, W (*where* A_s = *surface area of both the surfaces of plate*)

i.e. $Q := h_{avg} \cdot (2 \cdot L \cdot W)\,(T_s - T_a)$, W

i.e. $Q = 520.855$ **W** ***(total heat transfer from plate.)***

(vii) Temperature rise of air:

We have: $Q = m_{total} \cdot C_p \cdot \Delta T$

i.e. $$\Delta T := \frac{Q}{m_{total} \cdot C_p}$$

i.e. $\Delta T = 113.699$ **deg. C.**

Example 10.2. A furnace door, 1.5 m high and 1 m wide, is insulated from inside and has an outer surface temperature of 70°C. If the surrounding ambient air is at 30°C, calculate the steady state heat loss from the door.

Solution.

Data:

$L := 1.5$ m $W := 1.0$ m $T_s := 70$°C $T_a := 30$°C $g := 9.81$ m/s²

We need properties of air at film temperature $T_f = (70 + 30)/2$

$T_f := 50$°C (*film temperature*)

Properties of air at 50°C:

$\rho := 1.093$ kg/m³ $\nu := 17.95 \times 10^{-6}$ m²/s $Pr := 0.698$ $k := 0.02826$ W/(mK) $C_p := 1005$ J/kgK)

$$\beta := \frac{1}{(T_f + 273)}$$ (*coefficient of volume expansion...****Note that temperature must be in Kelvin***)

i.e. $\beta = 3.096 \times 10^{-3}$ 1/K

Grashoff number:

At $L = 1.5$ m: $$Gr_L := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2}$$

i.e. $Gr_L = 1.273 \times 10^{10}$

Rayleigh number:

$$Ra := Gr_L \cdot Pr$$

i.e. $$Ra = 8.882 \times 10^9$$

Then, applying Eq. 10.24, we get:

$$Nu := 0.13 \cdot Ra^{\frac{1}{3}} \quad (10^9 < R_a < 10^{12}...(10.24))$$

i.e. $$Nu = 269.227 \quad \textit{(Nusselt number)}$$

Therefore, $$h := \frac{Nu \cdot k}{L}\ \mathrm{W/(m^2K)} \quad \textit{(heat transfer coefficient)}$$

i.e. $$h = 5.072\ \mathrm{W/(m^2K)} \quad \textit{(heat transfer coefficient)}$$

Heat loss:

$$Q = h \cdot A_s \cdot (T_s - T_a),\ \mathrm{W} \quad \textit{(heat loss from outer surface)}$$

i.e. $$Q := h \cdot (L \cdot W) \cdot (T_s - T_a),\ \mathrm{W}$$

i.e. $$Q = 304.334\ \mathbf{W} \quad \textbf{\textit{(heat loss)}}$$

Alternatively, we can apply Eq. 10.26:

$$Nu := \frac{0.15 \cdot Ra^{\frac{1}{3}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{27}}} \quad (Ra > 10^9...(10.26))$$

i.e. $$Nu = 217.746$$

Therefore, $$h := \frac{Nu \cdot k}{L}\ \mathrm{W/(m^2K)} \quad \textit{(heat transfer coefficient)}$$

i.e. $$h = 4.102\ \mathrm{W/(m^2K)} \quad \textit{(heat transfer coefficient)}$$

And,

$$Q := h \cdot (L \cdot W) \cdot (T_s - T_a)\ \mathrm{W}$$

i.e. $$Q = 246.14\ \mathbf{W} \quad \textbf{(heat loss)}$$

Difference between the two values of Q obtained is about 19%

Example 10.3. In a nuclear reactor core, parallel vertical plates, each 2.5 *m* high and 1.5 *m* wide, heat liquid Bismuth by natural convection. Maximum temperature of the plates should not exceed 755°C and lowest allowable temperature of Bismuth is 320°C. Calculate the maximum heat dissipation from both sides of each plate.

Solution.

Data:

$L := 2.5$ m $\quad W := 1.5$ m $\quad T_s := 755$°C $\quad T_a := 320$°C $\quad g := 9.81\ \mathrm{m/s^2}$

We need properties of air at film temperature $T_f = (755 + 320)/2$

$$T_f := 537.5°\mathrm{C} \quad \textit{(film temperature)}$$

Properties of Bismuth at 538°C:

$\rho := 9739\ \mathrm{kg/m^3} \quad \nu := 1.08 \times 10^{-7}\ \mathrm{m^2/s} \quad Pr := 0.011 \quad k := 15.58\ \mathrm{W/(mK)}$

$C_p := 154.5\ \mathrm{J/(kgK)} \quad \beta := 0.126 \times 10^{-3}\ \mathrm{1/k}$

Note that we cannot put $\beta = 1/(T_f + 273)$, since Bismuth is a liquid, and not ideal gas. Instead, we should read the value of β from data tables.

Grashoff number:

At $L = 2.5$ m: $$Gr_L := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2}$$

i.e. $$Gr_L = 7.203 \times 10^{14}$$

Rayleigh number:

$$Ra := Gr_L \cdot Pr$$

i.e. $$Ra = 7.923 \times 10^{12}$$

Then, applying Eq. 10.27 we get:

$$Nu := \left[0.825 + \frac{0.387 \cdot Ra^{\frac{1}{6}}}{\left[1+\left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2 \qquad (Ra > 10^9 ...(10.27))$$

i.e. $Nu = 834.346$ *(Nusselt number)*

Therefore, $h := \frac{Nu \cdot k}{L}$ W/(m²K) *(heat transfer coefficient)*

i.e. $h = 5.2 \times 10^3$ W/(m²K) *(heat transfer coefficient)*

Heat loss:

$Q = h \cdot 2 \cdot A_s \cdot (T_s - T_a)$, W *(heat loss from both surfaces of plate)*

i.e. $Q := h \cdot 2 \cdot (L \cdot W) \cdot (T_s - T_a)$, W

i.e. $Q = 1.696 \times 10^7$ W *(heat loss.)*

i.e. $Q = 16.96$ MW *(heat loss from each plate.)*

Example 10.4. A vertical steel plate, 0.4 *m* × 0.4 *m* in size and 3 mm thick, at an uniform temperature of 180°C, is exposed to atmospheric air at 20°C. Find the approximate time required for the plate to cool to 30°C, if the heat transfer coefficient in natural convection for the vertical plate is given by: $h = 1.42 \times (\Delta T/L)^{1/4}$. For steel, $\rho = 7800$ kg/m³, $C_p = 473$ J/(kgK)

Solution.

Data:

$L := 0.4$ m $\quad W := 0.4$ m $\quad t := 0.003$ m $\quad \rho := 7800$ kg/m³ $\quad Ta := 20$°C $\quad Ts := 180$°C $\quad g := 9.81$ m/s²

$A := L \cdot W$ m² i.e. $A := 0.16$ m² $\quad C_p := 473$ J/(kgK)

At any instant, let the temperature of the plate be T. Then, we can write the heat balance:

Rate of decrease of enthalpy of the plate = rate of instantaneous heat transfer from plate by convection

i.e. $$-m \cdot C_p \frac{dT}{d\tau} = h \cdot (2 \cdot A) \cdot (T - Ta)$$ ((a)...*areas on both the sides of the plate lose heat by convection)*

where m is the mass of the plate

Put: $\theta = (T - T_a)$

Then, $$\frac{d\theta}{d\tau} = \frac{dT}{d\tau}$$

And Eq. a becomes:

$$\frac{-d\theta}{d\tau} = \frac{2 \cdot h \cdot A}{m \cdot C_p} \cdot \theta \qquad ...(b)$$

Now, mass of plate: $m := (L \cdot W \cdot t) \cdot \rho$ kg

i.e. $m = 3.744$ kg

Now, heat transfer coefficient: $h = 1.42 \cdot \left[\frac{(T - T_a)}{L}\right]^{\frac{1}{4}}$

i.e. $h = 1.78556\ \theta^{\frac{1}{4}}$, since $\theta = (T - T_a)$

Substituting in Eq. b:

$$\frac{-d\theta}{d\tau} = 3.22647 \times 10^{-4} \cdot \theta^{\frac{5}{4}} \qquad ...(c)$$

Integrating Eq. c:

$$4 \cdot \theta^{\frac{-1}{4}} = 3.22647 \times 10^{-4} \cdot \tau + C_1 \qquad ...(d)$$

where C_1 is the integration constant.

To find C_1, use the initial condition, i.e. at $\tau = 0$, $\theta = 180 - 20 = 160$°C:

i.e. $C_1 := 4.160^{\frac{-1}{4}}$

i.e. $C_1 = 1.12468$

Therefore Eq. *d* becomes

$$\theta^{\frac{-1}{4}} = 8.06618 \times 10^{-5} \cdot \tau + 0.28117 \qquad \text{...(e)}$$

Eq. e gives the temperature of the plate at any time τ.

Time required for the plate to reach 30°C:

i.e. $T := 30°C$

Therefore, $\theta := T - Ta$

i.e. $\theta = 10°C$

Then, from Eq. e:

$$\tau := \frac{\theta^{\frac{-1}{4}} - 0.28117}{8.06618 \times 10^{-5}}$$

i.e. $\tau = 3.486 \times 10^3$ s

i.e. $\tau = 0.968$ **hrs.**

Example 10.5. A vertical pipe, 15 cm OD, 1 *m* long, has a surface temperature of 90°C. If the surrounding air is at 30°C, what is the rate of heat loss by free convection per metre length of pipe?

(b) If the pipe is inclined to the vertical at an angle of 30 deg. during installation, how does the heat loss/m change?

Solution.

Data:

$L := 1.0$ m $\quad D := 0.15$ m $\quad T_a := 30°C \quad T_s := 90°C \quad g := 9.81$ m/s^2

Now, film temperature is (90 + 30)/2 = 60°C.

i.e. $T_f := 60°C$

Properties of air at 60°C:

$\nu := 18.97 \times 10^{-6}$ m^2/s $\quad Pr := 0.696 \quad k := 0.02896$ W/(mK)

$$\beta := \frac{1}{(T_f + 273)}$$ *(coefficient of volume expansion...Note that **Temperature must be in Kelvin***

i.e. $\beta = 3.003 \times 10^{-3}$ 1/K

Grashoff number:

$$Gr_L := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2}$$

i.e. $Gr_L = 4.912 \times 10^9$

$Ra_L := Gr_L \cdot Pr$

Rayleight number:

i.e. $Ra_L = 3.419 \times 10^9$

Now, to apply the vertical plate correlations for this case of a vertical cylinder, let us confirm the following condition:

$$\frac{D}{L} \geq \frac{34}{Ra_L^{\frac{1}{4}}} \qquad \text{...(10.28)}$$

$$\frac{D}{L} = 0.15 \quad \text{and,} \quad \frac{34}{Ra_L^{\frac{1}{4}}} = 0.141$$

Therefore, Eq. 10.28 is satisfied, and we can apply the vertical plate Eq. 10.24:

$$Nu := 0.13 \cdot Ra_L^{\frac{1}{3}} \qquad (10^9 < Ra < 10^{12}\text{...(10.24)})$$

i.e. $Nu = 195.836$ *(Nusselt number)*

Therefore, $h := Nu \cdot \frac{k}{L}$

i.e. $h = 5.671$ W/(m^{2}K) *(heat transfer coefficient)*

Heat loss/meter length of pipe:

$$Q := h\cdot(\pi\cdot D)\cdot(T_s - T_a) \text{ W/m}$$

i.e. $Q = 160.356$ **W/m.**

Alternatively:

We can use Eq. 10.27

$$Nu := \left[0.825 + \frac{0.387\cdot Ra^{\frac{1}{6}}}{\left[1+\left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2 \qquad (Re_L > 10^9...(10.27))$$

i.e. $Nu = 179.503$ *(compare with Nu = 195.836, obtained using Eq. 10.24)*

Therefore, $h := Nu\cdot\frac{k}{L}$

i.e. $h = 5.198 \text{ W/(m}^2\text{K)}$ *(heat transfer coefficient)*

and,

$$Q := h\cdot(\pi\cdot D)\cdot(T_s - T_a) \text{ W/m}$$

i.e. $Q = 146.981$ **W/m** ***(compare with 160.356 W/m obtained earlier.)***

(b) When the pipe is inclined at 30 deg. to vertical:

$\theta = 30$ deg.

But, while using Mathad, arguments to trigonometric functions must be in radians.

i.e. $\theta := 30\cdot\frac{\pi}{180}$ radians

i.e. $\theta = 0.524$ radians

We use $Nu := 0.13\cdot(Ra_L\cdot\cos(\theta))^{\frac{1}{3}}$

i.e. $Nu = 186.668$ *(Nusselt number)*

Therefore, $h := Nu\cdot\frac{k}{L}$

i.e. $h = 5.406 \text{ W/(m}^2\text{K)}$ *(heat transfer coefficient)*

Heat loss/metre length of pipe:

$$Q := h\cdot(\pi\cdot D)\cdot(T_s - T_a) \text{ W/m}$$

i.e. $Q = 152.849$ W/m.

10.5.4 Horizontal Plate at Constant Temperature T_s

Here, the characteristic length to be used in expressions for Nu and Gr is:

$$L_c = A/P$$

where, A is the surface area and P is the perimeter.

Property values are evaluated at film temperature, T_f.

(a) Upper surface of a hot plate (or, lower surface of a cold plate):

$$Nu = 0.54\, Ra^{\frac{1}{4}} \qquad (10^4 < Ra < 10^7...(10.34))$$

and,

$$Nu = 0.15\, Ra^{\frac{1}{3}} \qquad (10^7 < Ra < 10^{11}...(10.35))$$

(b) Lower surface of a hot plate (or upper surface of a cold plate):

$$Nu = 0.27\cdot Ra^{\frac{1}{4}} \qquad (10^5 < Ra < 10^{11}...(10.36))$$

Example 10.6. A hot, square plate, 50 cm × 50 cm, at 100°C is exposed to atmospheric air at 20°C. Find the heat loss from both the surfaces of the plate:

(i) if the plate is kept vertical

(ii) if the plate is kept horizontal.

Properties of air at mean temperature of 60°C are given below: $\rho = 1.06$ kg/m^3, $k = 0.028$ W/(mK), $\nu = 18.97 \times 10^{-6}$ m^2/s, $C_p = 1.008$ kJ/(kgK).

Following empirical relations can be used:

Case (i): $Nu = 0.13 \times (Gr.Pr)^{1/3}$

Case (ii): $Nu = 0.71 \times (Gr.Pr)^{1/4}$ for the upper surface, and

$Nu = 0.35 x (Gr.Pr)^{1/4}$ for the lower surface. (M.U. 1995)

Solution.

Data:

$L := 0.5$ m $\quad W := 0.5$ m $\quad T_s := 100°C \quad T_a := 20°C \quad g := 9.81$ m/s^2

We need properties of air at film temperature $T_f = (100 + 20)/2$

$$T_f := 60°C$$

Properties of air at 60°C

$\rho := 1.06$ kg/m^3 $\quad \nu := 18.97 \times 10^{-6}$ m^2/s $\quad \kappa := 0.028$ W/(mK) $\quad C_p := 1008$ J/(kgK) $\quad \mu := \nu \cdot \rho$

i.e. $\mu = 2.011 \times 10^{-5}$ kg/m.s $\quad Pr := \frac{C_p \cdot \mu}{k}$ $\quad$ i.e $Pr := 0.724$ $\quad \beta := \frac{1}{T_f + 273}$

i.e. $\beta = 3.003 \times 10^{-3}$ 1/K

Case 1: Plate held vertical:

Now, the characteristic length is the vertical side, L

$$Gr := \frac{L^3 \cdot g \cdot \beta \cdot (T_s - T_a)}{\nu^2}$$

i.e. $Gr = 8.816 \times 10^8$ *(Grashoff number)*

and, $Ra := Gr \cdot Pr$

i.e. $Ra = 5.926 \times 10^8$ *(Rayleigh number)*

Therefore,

$$Nu := 0.13 \cdot (Gr \cdot Pr)^{\frac{1}{3}}$$

i.e. $Nu = 109.194$ *(Nusselt number)*

and, $h := \frac{k \cdot Nu}{L}$

i.e. $h = 6.115$ W/(m^{2}K) *(heat transfer coefficient)*

Therefore, heat transferred:

$Q := h \cdot (2 \cdot L \cdot W) \cdot (Ts - Ta)$ W *(heat transfer from both surfaces)*

i.e. $Q = 244.594$ W ***(heat transfer from both surfaces)***

Case 2: Plate held horizontal:

Now, Characteristic length L_c = surface area of plate/Perimeter

$$L_c := \frac{L \cdot W}{2 \cdot (L + W)}$$

i.e. $L_c = 0.125$ m

Then, $Gr := \frac{L_c^3 \cdot g \cdot \beta \cdot (T_s - T_a)}{\nu^2}$

i.e. $Gr = 1.279 \times 10^7$ *(Grashoff number)*

Therefore, $Ra := Gr \cdot Pr$ *(Rayleigh number)*

i.e. $Ra = 9.259 \times 10^6$

For upper surface:

$$Nu_{upper} := 0.71 \cdot Ra^{\frac{1}{4}}$$

i.e. $Nu_{upper} = 39.166$ *(Nusselt number)*

and, $h_{upper} := \frac{Nu_{upper} \cdot k}{L_c}$ *(heat transfer coefficient)*

$h_{upper} = 8.773$ W/(m^{2}K) *(heat transfer coefficient for upper surface)*

$Q_{upper} := h_{upper} \cdot (L \cdot W) \cdot (T_s - T_a)$

i.e. $Q_{upper} = 175.462$ W *(heat transfer from upper surface)*

For lower surface:

$$Nu_{lower} := 0.35 \cdot Ra^{\frac{1}{4}}$$

i.e. $Nu_{lower} = 19.307$ *(Nusselt number)*

and, $$h_{lower} := \frac{Nu_{lower} \cdot k}{L_c}$$

i.e. $h_{lower} = 4.325\ W/(m^2K)$ *(heat transfer coefficient for lower surface)*

and, $Q_{lower} := h_{lower}\ (L \cdot W) \cdot (T_s - T_a)$

i.e. $Q_{lower} = 86.495\ W$ *(heat transfer from lower surface)*

Total heat transferred:

$$Q_{tot} := Q_{upper} + Q_{lower}$$

i.e. $Q_{tot} = 261.957\ W$ ***(Total heat transferred.)***

10.5.5 Horizontal Plate With Constant Heat Flux

Here, the characteristic length to be used in expressions for Nu and Gr is:

$$L_c = A/P$$

where, A is the surface area and P is the perimeter.

For a circle, $L_c = 0.9D$, and for rectangle, $L_c = (L + W)/2$.

All property values, except β, are evaluated at a temperature, T_e, defined by:

$$T_e = T_s - 0.25\ (T_s - T_a) \quad ...(10.37)$$

and, β is evaluated at T_a.

T_s is estimated from the basic relation:

$$h_{avg}\ (T_s - T_a) = q_s \quad ...(10.38)$$

(a) Upper surface of a hot plate (or lower surface of a cold plate):

$$Nu = 0.13 \cdot Ra^{\frac{1}{3}} \quad (Ra < 2 \times 10^8 ...(10.39))$$

and,

$$Nu = 0.16 \cdot Ra^{\frac{1}{3}} \quad (2 \times 10^8 < Ra < 10^{11} ...(10.40))$$

(b) For heated surface facing downward:

$$Nu = 0.58 \cdot Ra^{0.2} \quad (10^6 < Ra < 10^{11} ...(10.41))$$

As in the case of vertical plates with constant heat flux, in this case also, iteration will be required while solving problems.

Example 10.7. A horizontal metal plate, 0.5 *m* × 0.5 m, is exposed to sun and receives radiant energy at the rate 180 W/m^2. If the heat transfer from the plate occurs to the surrounding air at 20°C by free convection only, find the steady state temperature of the plate. Assume that the bottom of the plate is insulated.

Solution. The plate is subjected to constant heat flux. We do not know the surface temperature. So, we can assume either the surface temperature or the heat transfer coefficient to start with, proceed with the calculations, repeat if necessary.

Data:

$L := 0.5\ m \quad W := 0.5\ m \quad Q_s := 180\ W/m^2 \quad T_a := 20°C \quad g := 9.81\ m/s^2$

Let us assume the surface temperature to be 60°C. $T_s := 60°C$

This is consant flux condition. So, we need properties of air at T_e:

$$T_e := T_s - 0.25 \cdot (T_s - T_a)$$

i.e. $T_e = 50°C$

Properties of air at 50°C:

$\nu := 17.95 \times 10^{-6}\ m^2/s \quad k := 0.02826\ W/(mK) \quad Pr := 0.698 \quad \beta := \frac{1}{T_a + 273}$ i.e. $\beta = 3.413 \times 10^{-3}\ 1/K$

Now, for horizontal plate, Characteristic length L_c = surface area of plate/Perimeter

$$L_c := \frac{L \cdot W}{2 \cdot (L + W)}$$

i.e. $L_c = 0.125\ m$

Then, $Gr := \frac{L_c^3 \cdot g \cdot \beta \cdot (T_s - T_a)}{\nu^2}$

i.e. $Gr = 8.118 \times 10^6$ *(Grashoff number)*

Therefore, $Ra := Gr \cdot Pr$ *(Rayleigh number)*

i.e. $Ra = 5.667 \times 10^6$

Therefore, for upper surface of horizontal plate losing heat, we have:

$Nu := 0.13 \cdot Ra^{\frac{1}{3}}$ $(Ra < 2 \times 10^8 ...(10.39))$

i.e. $Nu = 23.177$ *(Nusselt number)*

and, $h := Nu \cdot \frac{k}{L_c}$ *(heat transfer coefficient)*

i.e. $h = 5.24\ W/(m^2K)$ *(heat transfer coefficient)*

Therefore, equating the heat received by plate to the heat transfer from the plate by convection:

$$q_s(L \cdot W) = h \cdot (L \cdot W) \cdot (T_s - T_a)$$

Therefore, $T_s := \frac{q_s \cdot (L \cdot W)}{h \cdot (L \cdot W)} + T_a$

i.e. $T_s = 54.353°C$

We had assumed T_s to be 60°C. So, let us repeat the calculations with T_s = 56°C

$$T_s := 56°C$$

We need properties of air at film temperature T_e

$$T_e := T_s - 0.25 \cdot (T_s - T_a)$$

i.e. $T_e = 47°C$

Properties of air at 47°C

$\nu := 17.7 \times 10^{-6}\ m^2/s$ *(kinematic viscosity)*

$k := 0.0275\ W/(mK)$ *(thermal conductivity)*

$Pr := 0.71$ *(Prandtl number)*

$$\beta := \frac{1}{T_a + 273}$$

i.e. $\beta = 3.413 \times 10^{-3}\ 1/K$

Then, $Gr := \frac{L_c^3 \cdot g \cdot \beta \cdot (T_s - T_a)}{\nu^2}$

i.e. $Gr = 7.514 \times 10^6$ *(Grashoff number)*

Therefore, $R_a := Gr \cdot Pr$ *(Rayleigh number)*

i.e. $Ra = 5.335 \times 10^6$

Therefore, for upper surface of horizontal plate losing heat, we have:

$Nu := 0.13 \cdot Ra$ $(Ra < 2 \times 10^8 ...(10.39))$

i.e. $Nu = 22.716$ *(Nusselt number)*

and, $h := Nu \cdot \frac{k}{L_c}$ *(heat transfer coefficient)*

i.e. $h = 4.997\ W/(m^2K)$ *(heat transfer coefficient)*

Therefore, equating the heat received by plate to the heat transfer from the plate by convection:

$$Q_s \cdot (L \cdot W) = h \cdot (L \cdot W) \cdot (T_s - T_a)$$

Therefore, $T_s := \frac{q_s \cdot (L \cdot W)}{h \cdot (L \cdot W)} + T_a$

i.e. $T_s = 56.018°C$

We had assumed T_s to be 56°C, whereas now, we got T_s = 56.018°C. This is in very good agreement.

Therefore, $T_s = 56°C$ ***(steady state surface temperature of plate.)***

10.5.6 Horizontal Cylinder At Constant Temperature

Here, D, diameter of the cylinder is the characteristic dimension.

For heat transfer from (or to) a horizontal cylinder, Morgan recommends following correlation for fluids with ($0.69 < Pr < 7$):

$$Nu = C \cdot Ra^n \quad ...(10.42)$$

where C and n are obtained from the following Table:

TABLE 10.2 Constants for use in Eq. 10.42

Ra	*C*	*n*
10^{-10}–10^{-2}	0.675	0.058
10^{-2}–10^{2}	1.02	0.148
10^{2}–10^{4}	0.85	0.188
10^{4}–10^{7}	0.48	0.25
10^{7}–10^{12}	0.125	0.333

Also, the following correlation of Churchill and Chu may be used for the complete range of Prandtl numbers: ($0 \le Pr \le \infty$) and for a wider range of Rayleigh numbers:

$$Nu = \left[0.60 + 0.387 \cdot \left[\frac{Ra}{\left[1 + \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}}}\right]^{\frac{1}{6}}\right]^2 \quad (10^{-5} < Ra < 10^{12} ...(10.43))$$

And, only for the laminar range:

$$Nu = 0.36 + \frac{0.518 \cdot Ra^{\frac{1}{4}}}{\left[1 + \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} \quad (10^{-6} < Ra < 10^{9} ...(10.44))$$

Properties in the above equations are evaluated at film temperature, D is the characteristic dimension. Churchill and Chu recommend that above two eqns. may be used for constant flux conditions too, with the temperature T_s being half way up the cylinder at the 90 deg. angle from bottom.

For thin wires: (D = 0.2 mm to 1 mm): Rayleigh number is usually very small and a film type of flow pattern is observed. Following correlation is used:

$$Nu_D = 1.18 \cdot (Ra_D)^{\frac{1}{8}} \quad (Ra < 500 ...(10.45))$$

Heat transfer from horizontal cylinders to **liquid metals** may be calculated from:

$$Nu_D = 0.53 \cdot (Gr_D \cdot Pr^2)^{\frac{1}{4}} \quad ...(10.46)$$

Example 10.8. A horizontal, steam pipe of 10 cm OD runs through a room where the ambient air is at 20°C. If the outside surface of the pipe is at 180°C, and the emissivity of the surface is 0.9, find out the total heat loss per metre length of pipe.

Solution. The pipe is horizontal and loses heat by natural convection as well as radiation. Diameter D is the characteristic dimension to calculate Rayleigh number.

Data:

$D := 0.1$ m $\quad L := 1.0$ m $\quad T_s := 180$°C $\quad T_a := 20$°C $\quad g := 9.81$ m/s^2 $\quad \varepsilon := 0.9$ $\quad \sigma := 5.67 \times 10^{-8}$ W/(m^{2}K^4)

We need properties of air at film temperature $T_f = (180 + 20)/2$

$$T_f := 100°C \quad \textit{(film temperature)}$$

Properties of air at 100°C:

$\rho := 0.946\ kg/m^3 \quad \nu := 23.02 \times 10^{-6}\ m^2/s \quad k := 0.03127\ W/(mK) \quad C_p := 1011.3\ J/(kgK) \quad Pr := 0.704$

$\beta := \dfrac{1}{T_f + 273}$ i.e. $\beta = 2.68 \times 10^{-3}\ 1/K$

Then, $$Gr := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot D^3}{\nu^2}$$

i.e. $$Gr = 7.94 \times 10^6 \quad \textit{(Grashoff number)}$$

Therefore, $$Ra := Gr \cdot Pr \quad \textit{(Rayleigh number)}$$

i.e. $$Ra = 5.59 \times 10^6$$

To find Nusselt number, we use Eq. 10.43:

$$Nu := \left[0.60 + 0.387 \cdot \left[\frac{Ra}{\left[1 + \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}}}\right]^{\frac{1}{6}}\right]^2 \quad (10^{-5} < Ra < 10^{12} ...(10.43))$$

i.e. $$Nu = 23.788 \quad \textit{(Nusselt number)}$$

And, $$h := Nu \cdot \frac{k}{D} \quad \textit{(heat transfer coefficient)}$$

i.e. $$h = 7.438\ W/(m^2K) \quad \textit{(heat transfer coefficient)}$$

Therefore, heat loss by natural convection:

$$Q_{conv} := h \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_a)\ W/m$$

i.e. $$Q_{conv} := 373.897\ W/m$$

And, heat loss by radiation:

Remember that, here, the temperatures must be in Kelvin.

$$Q_{rad} := \varepsilon \cdot (\pi \cdot D \cdot L) \cdot \sigma \cdot [(T_s + 273)^4 - (T_a + 273)^4]\ W/m$$

i.e. $$Q_{rad} = 556.947\ W/m$$

Therefore, total heat loss from pipe surface:

$$Q_{tot} := Q_{conv} + Q_{rad}\ W/m$$

i.e. $$Q_{tot} = 930.844\ \mathbf{W/m}$$

Note that in this type of problems, radiation heat loss is quite comparable to the natural convection heat loss and must, therefore, always be considered.

Alternatvely:

We can also use Eq. 10.42 to find out Nu, to determine the convection heat loss:

$$Nu = C \cdot Ra^n \quad ...(10.42)$$

Where constants C and n for $R_a = 5.59 \times 10^6$ are obtained from Table 10.2 as:

$$C := 0.48 \quad n := 0.25$$

Therefore, $$Nu := C \cdot Ra^n$$

i.e. $$Nu = 23.34 \quad \textit{(Nusselt number...compare with Nu = 23.788 obtained earlier)}$$

And, $$h := Nu \cdot \frac{k}{D} \quad \textit{(heat transfer coefficient)}$$

i.e. $$h = 7.298\ W/(m^2K) \quad \textit{(heat transfer coefficient...compare with h = 7.438 obtained earlier)}$$

Therefore, heat loss by natural convection:

$$Q_{conv} := h \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_a)\ W/m$$

i.e. $$Q_{conv} = 366.859\ W/m \quad \textit{(compares with 373.897 W/m, got earlier.)}$$

And, total heat loss from pipe surface:

$$Q_{tot} := Q_{conv} + Q_{rad}\ \text{W/m}$$

i.e. $Q_{tot} = 923.806$ **W/m.**

Example 10.9. A tank contains water at 15°C. The water is heated by passing steam through a pipe placed in water. The pipe is 60 cm long and 4 cm in diameter and its surface is maintained at 85°C. Find the heat loss from the pipe if:

(i) the pipe is kept horizontal

(ii) the pipe is kept vertical.

Following empirical relations may be used: $Nu = C.(Gr.Pr)^m$, where

$C = 0.53$ and $m = 0.25$ when $10^4 < Gr.Pr < 10^9$, and

$C = 0.13$ and $m = 1/3$ when $Gr.Pr > 10^9$. (M.U., 1998)

Following data may be used:

Properties of water at average temperature of 50°C are:

$T_f := 50°C$ $\quad \rho := 988\ \text{kg/m}^3$ $\quad \nu := 5.56 \times 10^{-7}\ \text{m}^2\text{/s}$ $\quad C_p := 4178\ \text{J/(kgK)}$

$k := 0.647\ \text{W/(mK)}$ $\quad \beta := 5.1 \times 10^{-4}\ 1/\text{K}$

Other data:

$L := 0.6\ \text{m}$ $\quad D := 0.04\ \text{m}$ $\quad T_s := 85°C$ $\quad T_a := 15°C$ $\quad g := 9.81\ \text{m/s}^2$ $\quad \mu := \nu\cdot\rho$ i.e. $\mu = 5.493 \times 10^{-4}\ \text{kg/(ms)}$

$Pr := \dfrac{C_p\cdot\mu}{k}$ i.e. $Pr := 3.547$

Case 1: Pipe held horizontal:

Diameter D is the characteristic dimension to calculate Ra.

$$A := \pi\cdot D\cdot L\ \text{m}^2 \qquad \textit{(surface area)}$$

i.e. $A = 0.075\ \text{m}^2$ *(surface area)*

And, $Gr := \dfrac{D^3\cdot g\cdot\beta\cdot(T_s - T_a)}{\nu^2}$

$Gr = 7.25 \times 10^7$ *(Grashoff number)*

And, $Ra := Gr\cdot Pr$

i.e. $Ra = 2.572 \times 10^8$ *(Rayleigh number)*

Then, we have:

$Nu := 0.53\cdot Ra^{0.25}$

i.e. $Nu = 67.118$ *(Nusselt number)*

and, $h := \dfrac{Nu\cdot k}{D}$

i.e. $h = 1.086 \times 10^3\ \text{W/(m}^2\text{K)}$ *(heat transfer coefficient)*

And heat transfer is given by:

$Q_{horizl} := h\cdot A\cdot(T_s - T_a)$

i.e. $Q_{horizl} = 5.73 \times 10^3\ \text{W}$

Case 2: Pipe held vertical:

Now, the length L is the characteristic dimension to calculate Ra.

We have: $Gr := \dfrac{L^3\cdot g\cdot\beta\cdot(T_s - T_a)}{\nu^2}$

i.e. $Gr = 2.447 \times 10^{11}$

and, $Ra := Gr\cdot Pr$

i.e. $Ra = 8.68 \times 10^{11}$

And, $Nu := 0.13\cdot Ra$

i.e. $Nu = 1.24 \times 10^3$ *(Nusselt number)*

and, $h := \dfrac{Nu\cdot k}{L}$

i.e. $h = 1.337 \times 10^3\ \text{W/(m}^2\text{K)}$ *(heat transfer coefficient.)*

And heat transfer is given by:

$Q_{vert} := h\cdot A\cdot(T_s - T_a)$

i.e. $Q_{vert} = 7.058 \times 10^3\ \text{W}$

Example 10.10. A fine wire of 0.2 mm diameter is maintained at a constant temperature of 64°C by an electric current. The wire is exposed to air at 1 bar and 10°C. Calculate the electric power necessary to maintain the wire temperature if the length of wire is 1 m.

Solution.

Data:

$L := 1.0$ m $\quad D := 2 \times 10^{-4}$ m $\quad T_s := 64°C \quad T_a := 10°C \quad g := 9.81$ m/s^2

Properties of air at film temperature of (64 + 10)/2 = 37°C are:

$T_f := 37°C \quad \rho := 1.143$ kg/m^3 $\quad \nu := 16.7 \times 10^{-6}$ m^2/s $\quad C_p := 1006$ J/(kgK) $\quad k := 0.0268$ W/(mK)

$Pr := 0.711 \quad \beta := \frac{1}{T_f + 273}$ 1/K i.e. $\beta = 3.226 \times 10^{-3}$ 1/K

Diameter D is the characteristic dimension to calculate Ra.

$$A := \pi \cdot D \cdot L \text{ m}^2 \qquad \textit{(surface area)}$$

i.e. $$A = 6.283 \times 10^{-4} \text{ m}^2 \qquad \textit{(surface area)}$$

And, $$Gr := \frac{D^3 \cdot g \cdot \beta \cdot (T_s - T_a)}{\nu^2}$$

$$Gr = 0.049018 \qquad \textit{(Grashoff number)}$$

And, $$Ra := Gr \cdot Pr$$

i.e. $$Ra = 0.035 \qquad \textit{(Rayleigh number)}$$

Then, applying Eq. 10.45, we get:

$$Nu_D := 1.18 \cdot (Ra)^{\frac{1}{8}} \qquad (Ra < 500 \ldots (10.45))$$

i.e. $$Nu_D = 0.776 \qquad \textit{(Nusselt number)}$$

and, $$h := Nu_D \cdot \frac{k}{D} \qquad \textit{(heat transfer coefficient)}$$

i.e. $$h = 103.936 \text{ W/(m}^2\text{K)} \qquad \textit{(heat transfer coefficient)}$$

Therefore, rate of heat loss from wire;

$$Q := h \cdot A \cdot (T_s - T_a) \text{ W/m}$$

i.e. $$Q = 3.526 \text{ W/m}$$

i.e. Power required to maintain the surface temperature at 64°C = 3.526 W

Alternatively:

We can use Eq. 10.42:

$$Nu = C \cdot Ra^n \qquad \ldots(10.42)$$

Where constants C and n are obtained from the Table, corresponding to $Ra = 0.035$:

$$C := 1.02 \text{ and } n = 0.148$$

Then,

$$Nu := C \cdot Ra^n$$

i.e. $$Nu = 0.621 \qquad \textit{(Nusselt number)}$$

and, $$h := Nu \cdot \frac{k}{D} \qquad \textit{(heat transfer coefficient)}$$

i.e. $$h = 83.168 \text{ W/(m}^2\text{K)} \qquad \textit{(compare this with } h = 103.936 \textit{ obtained earlier.)}$$

Therefore, rate of heat loss from wire:

$$Q := h \cdot A \cdot (T_s - T_a) \text{ W/m}$$

i.e. $$Q = 2.822 \text{ W/m} \qquad \textit{(compare this with } Q = 3.526 \textit{ W/m obtained earlier.)}$$

10.5.7 Free Convection From Spheres

Sphere diameter D is the characteristic dimension. Yuge recommends following correlation for average Nusselt number for free convection between a sphere and air. Properties are evaluated at the film temperature.

$$Nu = 2 + 0.43 \cdot (Ra)^{\frac{1}{4}} \qquad (1 < Ra < 10^5, Pr = 1 \ldots (10.47))$$

For higher range of Ra:

$$Nu = 2 + 0.50 \cdot (Ra)^{\frac{1}{4}} \qquad (3 \times 10^5 < Ra < 8 \times 10^8 \ldots (10.48))$$

Example 10.11. A sphere of 25 mm diameter, with its surface temperature at 100°C, is kept in still air at a temperature of 20°C. Determine the rate of convective heat loss.

Solution.

Data:

$D := 25 \times 10^{-3}$ m $\quad T_s := 100°C \quad T_a := 20°C \quad g := 9.81$ m/s²

Properties of air at film temperature of (100 + 20)/2 = 60°C are:

$T_f := 60°C \quad \nu := 18.97 \times 10^{-6}$ m²/s $\quad k := 0.02896$ W/(mK) $\quad Pr := 0.696 \quad \beta := \dfrac{1}{T_f + 273}$ 1/K

i.e. $\beta = 3.003 \times 10^{-3}$ 1/K

Diameter D is the characteristic dimension to calculate Ra.

$A := \pi \cdot D^2$ m² (*surface area of sphere*)

i.e. $A = 1.96 \times 10^{-3}$ m² (*surface area*)

And, $$Gr := \frac{D^3 \cdot g \cdot \beta \cdot (T_s - T_a)}{\nu^2}$$

$Gr = 1.023 \times 10^5$ (*Grashoff number*)

And, $Ra := Gr \cdot Pr$

i.e. $Ra = 7.122 \times 10^4$ (*Rayleigh number*)

Then, using Eq. 10.47:

$Nu := 2 + 0.43 \cdot (Ra)^{1/4}$ ($1 < Re < 10^5$, $Pr = 1$...(10.47))

i.e. $Nu = 9.025$ (*Nusselt number*)

and, $h := Nu \cdot \dfrac{k}{D}$ (*heat transfer coefficient*)

i.e. $h = 10.454$ W/(m²K) (*heat transfer coefficient*)

Therefore, rate of heat loss from sphere:

$Q := h \cdot A \cdot (T_s - T_a)$ W

i.e. $Q = 1.642$ W

10.5.8 Free Convection From Rectangular Blocks and Short Cylinders

Here, characteristic length L is defined as:

$$L = \frac{L_H \cdot L_V}{L_H + L_V} \quad ...(10.49a)$$

where L_H is the longer of the two horizontal dimensions and L_V is the vertical dimension. Based on this characteristic length, the heat transfer correlation is:

$$Nu_L = 0.55 \cdot (Ra_L)^{\frac{1}{4}} \quad (10^4 < Ra_L < 10^9 ...(10.49))$$

For short cylinders ($D = H$):

$$Nu = 0.775 \cdot (Ra)^{0.208} \quad ...(10.50)$$

Example 10.12. A ceramic block is of 0.3 *m* × 0.2 *m* section and is 0.3 *m* in height. Surface temperature of the block is 380°C. If it is exposed to air at 20°C, determine the rate of convective heat loss.

Solution.

Data:

$L_H := 0.3$ m $\quad L_V := 0.3$ m $\quad T_s := 380°C \quad T_a := 20°C \quad g := 9.81$ m/s²

Properties of air at film temperature of (380 + 20)/2 = 200°C are:

$T_f := 200°C \quad \nu := 34.57 \times 10^{-6}$ m²/s $\quad k := 0.03781$ W/(mK) $\quad Pr := 0.699 \quad \beta := \dfrac{1}{T_f + 273}$ 1/K

i.e. $\beta = 2.114 \times 10^{-3}$ 1/K

The characteristic dimension to calculate Ra, is:

$$L := \frac{L_H \cdot L_V}{L_H + L_V}$$

i.e. $L = 0.15$ m

Also, $A := 2 \cdot (0.3 \cdot 0.3) + 2 \cdot (0.2 \cdot 0.3) + 2 \cdot (0.3 \cdot 0.2)$ m² (*surface area of the block*) i.e. $A = 0.42\ m^2$ *(surface area)*

And, $Gr := \dfrac{L^3 \cdot g \cdot \beta \cdot (T_s - T_a)}{v^2}$

$Gr = 2.109 \times 10^7$ *(Grashoff number)*

and, $Ra := Gr \cdot Pr$

i.e. $Ra = 1.474 \times 10^7$ *(Rayleigh number)*

Then using Eq. 10.49 we get:

$$Nu := 0.55 \cdot (Ra)^{\frac{1}{4}}$$ $(10^4 < Ra < 10^9$...(10.49)

i.e. $Nu = 34.078$ *(Nusselt number)*

and, $h := Nu \cdot \dfrac{k}{L}$ *(heat transfer coefficient)*

i.e. $h = 8.59\ W/(m^2K)$ *(heat transfer coefficient)*

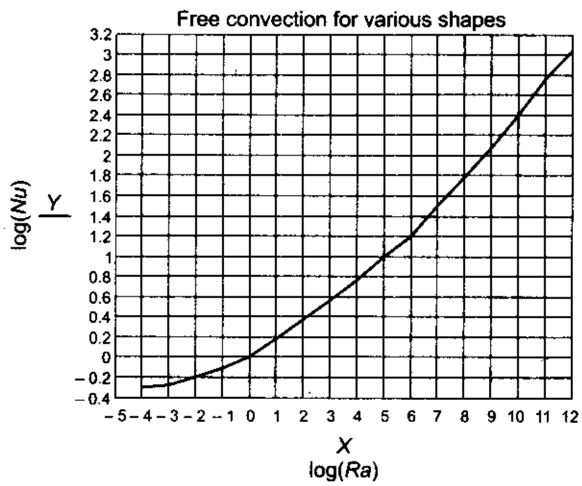

FIGURE Example 10.12 Graph of log(*Ra*) vs. log(*Nu*) for various shapes in free convection with various fluids

Therefore, rate of heat loss from the block:

$Q := h \cdot A \cdot (T_s - T_a)$ W

i.e. $Q = 1.299 \times 10^3$ **W**

Alternatively:

Based on experimental data for vertical plates, vertical cylinders, horizontal cylinders, spheres and blocks to various fluids such as air, water, alcohol and oil, King has drawn the following curve of log(*Ra*) vs. log(*Nu*). Here, fluid properties are evaluated at the film temperature and the characteristic dimension (*L*) to be taken to determine *Ra* and *Nu* are: for a vertical plate *L* is the height, and for long, horizontal cylinder, *L* is the diameter, and for a short cylinder or block, $1/L = (1/L_V) + (1/L_H)$. For a sphere, *radius* is the characteristic dimension.

Above Fig. Example 10.12 is expected to give fair estimate of convection coefficient for objects other than horizontal cylinder and plate. This figure can also be used for more common shapes when the Rayleigh number is outside the range of the specific correlation for that shape.

Now, in this problem: $Ra = 1.474 \times 10^7$

Therefore, $\log(Ra) = 7.168$

Using this value in the x-axis of above Fig. Example 10.12 we get:

$\log(Nu) = 1.55$

i.e. $Nu = 10^{1.55}$

i.e. $Nu := 35.481$

and, $h := Nu \cdot \frac{k}{L}$ *(heat transfer coefficient)*

i.e. $h = 8.944\ W/(m^2K)$ *(heat transfer coefficient)*

Compare this value of h with $h = 8.59\ W/(m^2K)$, got earlier.

Therefore, rate of heat loss from the block:

$$Q := h \cdot A \cdot (T_s - T_a)\ W$$

i.e. $Q = 1.352 \times 10^3\ W$

Compare this value of Q with $Q = 1299$ W, obtained earlier.

10.5.9 Simplified Equations For Air

Since air is the common fluid in most of the free convection problems encountered in practice, it is useful to have simplified relations for those situations:

TABLE 10.3 Simplified equations for free convection to air at atmospheric pressure (constant wall temp.)

Surface	Laminar $10^4 < (Gr.Pr) < 10^9$	Turbulent $(Gr.Pr) > 10^9$
Vertical plate or cylinder	$h = 1.42 \cdot \left(\frac{\Delta T}{L}\right)^{\frac{1}{4}}$	$h = 1.31 \cdot (\Delta T)^{\frac{1}{3}}$
Horizontal cylinder	$h = 1.32 \cdot \left(\frac{\Delta T}{D}\right)^{\frac{1}{4}}$	$h = 1.24 \cdot (\Delta T)^{\frac{1}{3}}$
Horizontal plate: Heated plate facing upward, or cooled plate facing downward	$h = 1.32 \cdot \left(\frac{\Delta T}{L}\right)^{\frac{1}{4}}$	$h = 1.52 \cdot (\Delta T)^{\frac{1}{3}}$
Heated plate facing downward, or cooled plate facing upward	$h = 0.59 \cdot \left(\frac{\Delta T}{L}\right)^{\frac{1}{4}}$ where h = heat transfer coefficient, L = vertical or horizontal dimension, D = diameter, and $\Delta T = T_s - T_a$	
Spheres	$h = [2 + 0.392 \cdot Gr_D^{1/4}] \cdot \frac{k}{D}$ for $1 < Gr_D < 10^5$	

For pressures other than atmospheric, multiply the RHS of above expressions as below, where p is in bar:

Laminar: $\left(\frac{p}{1.0132}\right)^{\frac{1}{2}}$

Turbulent $\left(\frac{p}{1.0132}\right)^{\frac{2}{3}}$

10.5.10 Free Convection In Enclosed Spaces

Enclosed spaces may be formed by horizontal plates or vertical plates; also, enclosed spaces may be filled with air or any other fluids. Typical example is a double-plane window, or a vacuum flask or a cryogenic container involving concentric cylinders or spheres. Correlations for convection heat transfer for such situations are given below.

Fig. 10.3 shows the horizontal and vertical enclosures and the nomenclature used. Here, the space between the plates, 'b' is the characteristic dimension. Properties are evaluated at the average of two plate temperatures.

So, now, Grashoff number for the enclosure is defined as:

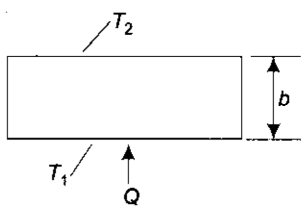

FIGURE 10.3 (A) Free convection in a horizontal Enclosure ($T_1 > T_2$)

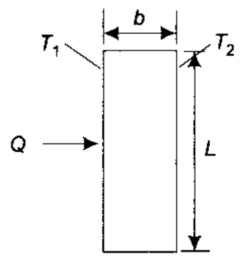

FIGURE 10.3 (B) Free convection in a vertical Enclosure ($T_1 > T_2$)

$$Gr_b = \frac{g \cdot \beta \cdot (T_1 - T_2) \cdot b^3}{\nu^2} \quad \text{...(10.51)}$$

and, the Rayleigh number for the enclosure is:

$$R_{ab} = \frac{g \cdot \beta \cdot (T_1 - T_2) \cdot b^3}{\nu \cdot \alpha} \quad \text{...(10.52)}$$

For Horizontal enclosure:

For air:

Average Nusselt number (based on plate spacing 'b') is given by Jakob:

$$Nu = 0.195 \cdot Gr^{\frac{1}{4}} \quad (10^4 < Gr < 3.7 \times 10^5 \text{...(10.53)})$$

And, $$Nu = 0.068 \cdot Gr^{\frac{1}{3}} \quad (3.7 \times 10^5 < Gr < 10^7 \text{...(10.54)})$$

And, for $Gr < 1700$, we have $Nu = 1$.

For liquids (water, silicone oils and mercury), equation suggested by Globe and Dropkin:

$$Nu = 0.069 \cdot Ra^{\frac{1}{3}} \cdot Pr^{0.074} \quad (1.5 \times 10^5 < Ra < 10^9 \text{...(10.55)})$$

Here also, the space between the plates, 'b' is the characteristic dimension. Properties are evaluated at the average of two plate temperatures.

For Vertical enclosure:

For Air:

For Gr (based on plate spacing 'b') < 1700, we have $Nu = 1$.

Jakob has given following correlations:

$$\frac{k_{eff}}{k} = Nu = \frac{0.18 \cdot Gr^{\frac{1}{4}}}{\left(\frac{L}{b}\right)^{\frac{1}{9}}} \quad (2 \times 10^4 < Gr < 2 \times 10^5 \text{...(10.56)})$$

where k_{eff} = effective thermal conductivity

and, $$\frac{k_{eff}}{k} = Nu = \frac{0.065 \cdot Gr^{\frac{1}{4}}}{\left(\frac{L}{b}\right)^{\frac{1}{9}}} \quad (2 \times 10^5 < Gr < 10^7 \text{...(10.57)})$$

Note that for above two relations, aspect ratio, $L/b > 3$.

If $L/b < 3$, each vertical surface is treated independently.

If the enclosed vertical layer contains fluids with Prandtl numbers between 3 and 30,000, following correlation due to Emery and Chu may be used:

$$Nu = 1 \qquad \textit{(for } Ra < 1000\text{...(10.58))}$$

And,

$$Nu = \frac{0.28 \cdot Ra^{\frac{1}{4}}}{\left(\frac{L}{b}\right)^{\frac{1}{4}}} \qquad \textit{(for } 1000 < Ra < 10^7\text{...(10.59))}$$

Layer thickness 'b' is the characteristic dimension used in Nu and Ra.

Example 10.13. Air at 2 bar pressure is contained between two horizontal panels separated by a distance of 20 mm. The lower panel is at a temperature of 70°C and the upper panel is at 30°C. Calculate the heat transfer rate by free convection per sq. m. of the panel surface.

Solution.

Data:

$L := 0.02$ m $\quad T_1 := 70°C \quad T_2 := 30°C \quad g := 9.81$ m/s^2 $\quad P := 2 \times 10^5$ Pa $\quad R := 287$ J/kgK

We need properties of air at film temperature $T_f = (70 + 30)/2$

$$T_f := 50°C \qquad \textit{(average temperature)}$$

Properties of air at 50°C:

Note that only density changes with pressure and μ, k and C_p do not change much; however, $\nu = \mu/\rho$, and this changes with pressure.

$$\rho := \frac{P}{R \cdot (T_f + 273)} \text{ kg/m}^3 \qquad \textit{(density of air at 2 bar pressure)}$$

i.e. $\rho = 2.157$ kg/m^3 *(density of air at 2 bar pressure)*

$\mu := 19.57 \times 10^{-6}$ kg/ms *(dynamic viscosity)*

Therefore, $$\nu := \frac{\mu}{\rho}$$

i.e. $\nu = 9.07 \times 10^{-6}$ m^2/s *(kinematic viscosity at 2 bar pressure)*

$k := 0.02781$ W/(mK) *(thermal conductivity)*

$Pr := 0.709$ *(Prandtl number)*

$$\beta := \frac{1}{T_f + 273}$$

i.e. $\beta = 3.096 \times 10^{-3}$ 1/K

Then, $$Gr := \frac{g \cdot \beta \cdot (T_1 - T_2) \cdot L^3}{\nu^2}$$

i.e. $Gr = 1.181 \times 10^5$ *(Grashoff number)*

Then, using Eq. 10.53, we get:

$$Nu := 0.195 \cdot Gr^{\frac{1}{4}} \qquad (10^4 < Gr < 3.7 \times 10^5\text{...(10.53))}$$

i.e. $Nu = 3.615$ *(Nusselt number)*

Then, heat flux across the gap is computed from:

$$Nu = \frac{q \cdot L}{k \cdot (T_1 - T_2)}$$

i.e. $$q := \frac{Nu \cdot k \cdot (T_1 - T_2)}{L} \qquad \textit{...heat flux across the gap}$$

i.e. $q = 201.07$ **W/m^2** ***...heat flux across the gap***

Example 10.14. Air gap between the two glass panels of a double-pane window (0.8 *m* wide × 1.5 *m* high) is 2 cms. If the two glass surfaces are at 20°C and 0°C, determine the rate of heat transfer through the window.

Solution.

Data:

$L := 1.5$ m $\quad W := 0.8$ m $\quad b := 0.02$ m $\quad T_1 := 20°C \quad T_2 := 0°C \quad g := 9.81$ m/s^2

We need properties of air at film temperature $T_f = (20 + 0)/2$

$$T_f := 10°C \quad \textit{(average temperature)}$$

Properties of air at 10°C:

$\nu := 14.19 \times 10^{-6}\ m^2/s$ $\quad k := 0.02487\ W/(mK)$ $\quad Pr := 0.716$ $\quad \beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.534 \times 10^{-3}\ 1/K$

Remember that here, the distance between panels 'b' is the characteristic dimension.

Then, $$Gr := \frac{g \cdot \beta \cdot (T_1 - T_2) \cdot b^3}{\nu^2}$$

i.e. $$Gr = 2.754 \times 10^4 \quad \textit{(Grashoff number)}$$

And, $$\frac{L}{b} = 75 > 3 \quad \textit{(condition is satisfied.)}$$

Then, using Eq. 10.56 we get:

$$Nu := \frac{0.18 \cdot Gr^{\frac{1}{4}}}{\left(\frac{L}{b}\right)^{\frac{1}{9}}} \quad (2 \times 10^4 < Gr < 2 \times 10^5 ...(10.56))$$

i.e. $$Nu = 1.435 \quad \textit{(Nusselt number)}$$

Therefore, $$k_{eff} := Nu \cdot k$$

i.e. $$k_{eff} = 0.036\ W/(mK) \quad \textit{(effective thermal conductivity)}$$

Then, heat flux across the gap is computed from:

$$Nu := \frac{q \cdot b}{k \cdot (T_1 - T_2)}$$

i.e. $$q = \frac{Nu \cdot k \cdot (T_1 - T_2)}{b} \quad \textit{(heat flux across the gap)}$$

Defining 'effective thermal conductivity', we can also write the above relation as:

$$k_{eff} := Nu \cdot k \quad \textit{(effective thermal conductivity)}$$

i.e. $$k_{eff} = 0.036\ W/(mK) \quad \textit{(effective thermal conductivity)}$$

and, $$q := \frac{k_{eff} \cdot (T_1 - T_2)}{b} \quad \textit{(heat flux across the gap)}$$

i.e. $$q = 35.696\ W/m^2 \quad \textit{(heat flux across the gap)}$$

Therefore, $$Q := q \cdot (L \cdot W)\ W \quad \textit{(heat transfer rate)}$$

i.e. $$Q = 42.835\ \mathbf{W} \quad \textbf{\textit{(heat transfer rate)}}$$

10.5.11 Free Convection In Inclined Spaces

This situation is encountered in flat plate solar collectors and double-glazed windows. Fig. 10.4 shows the nomenclature for the relations given below.

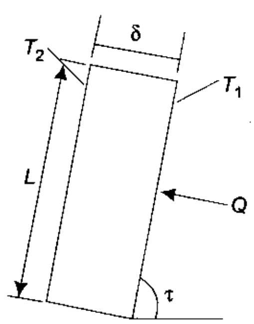

FIGURE 10.4 Free convection in inclined, enclosed space ($T_1 > T_2$)

This configuration has been investigated for large aspect ratios ($L/\delta < 12$) by Hollands et.al. Following equation correlated experimental data at tilt angles τ less than 70 deg.:

$$Nu_L = 1 + 1.44 \cdot \left(1 - \frac{1708}{Ra_L \cdot \cos(\tau)}\right) \cdot \left[1 - \frac{1708 \cdot (\sin(1.8 \cdot \tau))^{1.6}}{Ra_L \cdot \cos(\tau)}\right] + \left(\left[\frac{Ra_L \cdot \cos(\tau)}{5830}\right]^{\frac{1}{3}} - 1\right) \quad ...(10.60)$$

If the quantity in the first bracket and the last bracket is negative, then it must be set equal to zero.

For tilt angles between 70 deg. and 90 deg. Catton recommends that the Nusselt number for a vertical enclosure (τ = 90 deg.) be multiplied by $(\sin \tau)^{1/4}$

i.e.

$$Nu_L(\tau) = Nu_L(\tau = 90)\cdot(\sin(\tau)) \qquad ...(10.61)$$

Example 10.15. In a solar flat plate collector, the plate is of size 1 *m* × 1 *m* and is at a temperature of 140°C. The glass cover plate is at a distance of 8 cm from the collector surface and its temperature is 40°C. Space in between contains air at 1 atm. If the collector plate is inclined to the horizontal at 20 deg., determine the heat transfer coefficient.

Solution.

Data:

$L := 1.0$ m $W := 1.0$ m $b := 0.08$ m $T_1 := 140°C$ $T_2 := 40°C$ $\tau := 20$ deg (*angle of tilt (to horizontal)*)

But, while using Mathad, arguments for trigonometric functions must be in radians: So,

$$\tau := 20\cdot\frac{\pi}{180} \qquad \textit{(radians...angle of tilt (to horizontal))}$$

$$g := 9.81 \text{ m/s}^2 \qquad \textit{(acceleration due to gravity)}$$

We need properties of air at average temperature $T_f = (140 + 40)/2$

$$T_f := 90°C \qquad \textit{(average temperature)}$$

Properties of air at 90°C:

$$\nu := 21.96 \times 10^{-6} \text{ m}^2/\text{s} \qquad \textit{(kinematic viscosity)}$$

$$k := 0.03059 \text{ W/(mK)} \qquad \textit{(thermal conductivity)}$$

$$Pr := 0.705 \qquad \textit{(Prandtl number)}$$

$$\beta := \frac{1}{T_f + 273}$$

i.e. $\beta = 2.755 \times 10^{-3}$ 1/K

Remember that here, the height of panels '*L*' is the characteristic dimension.

And, $$\frac{L}{b} = 12.5 > 12 \qquad \textit{(condition is satisfied.)}$$

Then, $$Gr_L := \frac{g\cdot\beta\cdot(T_1 - T_2)\cdot L^3}{\nu^2}$$

i.e. $Gr_L = 5.604 \times 10^9$ *(Grashoff number)*

and, $Ra_L := Gr_L\cdot Pr$ *(Rayleigh number)*

i.e. $Ra_L = 3.951 \times 10^9$ *(Rayleigh number)*

Then, using Eq. (10.60), we get:

$$Nu_L := 1 + 1.44\cdot\left(1 - \frac{1708}{Ra_L\cdot\cos(\tau)}\right)\cdot\left[1 - \frac{1708\cdot(\sin(1.8\cdot\tau))^{1.6}}{Ra_L\cdot\cos(\tau)}\right] + \left[\left(\frac{Ra_L\cdot\cos(\tau)}{5830}\right)^{\frac{1}{3}} - 1\right] \qquad ...(10.60)$$

We have: $$\left(1 - \frac{1708}{Ra_L\cdot\cos(\tau)}\right) = 1 \qquad \textit{(not negative)}$$

and, $$\left[\left(\frac{Ra_L\cdot\cos(\tau)}{5830}\right)^{\frac{1}{3}} - 1\right] = 85.034 \qquad \textit{(not negative)}$$

Note: If the above two terms are negative, then they must be set equal to zero.

Therefore, $Nu_L = 87.474$ *(Nusselt number)*

i.e. $$h := \frac{Nu_L\cdot k}{L} \text{ ...W/(m}^2\text{K)} \qquad \textit{(heat transfer coefficient)}$$

i.e. $h = 2.676$ **W/(m²K)** ***(heat transfer coefficient.)***

10.5.12 Natural Convection Inside Spherical Cavities

Diameter *D* is the characteristic dimension. Following relation is recommended:

$$\frac{D\cdot h_{avg}}{k} = C\cdot(Gr_D\cdot Pr)^n \qquad ...(10.62)$$

where C and n are taken from table below:

$Gr_D Pr$	C	n
$10^4 - 10^9$	0.59	1/4
$10^9 - 10^{12}$	0.13	1/3

10.5.13 Natural Convection Inside Concentric Cylinders and Spheres

Free convection in enclosures formed between concentric cylinders and concentric spheres when the gap is filled with various fluids such as air, water and oils have been correlated by Raithby and Hollands.

See Fig. 10.5. Here, D_i and D_o are the inside and outside diameters of the long cylinders or spheres; T_i and T_o are the corresponding temperatures. L is the length of long cylinders, and 'b' is the gap or thickness of the enclosed fluid layer (i.e. $b = [D_o - D_i]/2$). Procedure is to find out an effective thermal conductivity and then determine the heat transfer as if by pure conduction, using this effective thermal conductivity.

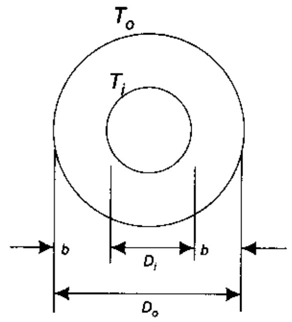

FIGURE 10.5 Free convection in an enclosure between long, concentric cylinders and spheres ($T_i > T_o$)

Concentric cylindrical annuli:

$$\frac{Q}{L} = \frac{2 \cdot \pi \cdot k_{eff} \cdot (T_i - T_o)}{\ln\left(\frac{D_o}{D_i}\right)} \quad ...(10.63)$$

$$\frac{k_{eff}}{k} = 0.386 \cdot \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} \cdot Ra_{cc}^{\frac{1}{4}} \quad (100 < Ra_{cc} < 10^7 ...(10.64))$$

And,

$$Ra_{cc} = \frac{\left(\ln\left(\frac{D_o}{D_i}\right)\right)^4 \cdot Ra_b}{b^3 \cdot \left[\frac{1}{D_i^{\frac{3}{5}}} + \frac{1}{D_o^{\frac{3}{5}}}\right]^5} \quad ...(10.65)$$

Concentric spherical annuli:

$$Q = \pi \cdot k_{eff} \cdot \frac{D_i \cdot D_o}{b} \cdot (T_i - T_o) \quad ...(10.66)$$

$$\frac{k_{eff}}{k} = 0.74 \cdot \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} \cdot Ra_{cs}^{\frac{1}{4}} \quad (10^2 < Ra_{cs} < 10^4 ...(10.67))$$

And,

$$Ra_{cs} = \frac{b \cdot Ra_b}{D_o^4 \cdot D_i^4 \cdot \left[\frac{1}{D_i^{\frac{7}{5}}} + \frac{1}{D_o^{\frac{7}{5}}}\right]^5} \quad ...(10.68)$$

In both Eqs. 10.65 and 10.68, Rayleigh number (Ra_b) is based on the thickness 'b' of the annular fluid layer. Further, fluid properties are to be evaluated at the average of T_i and T_o, and Eqs. 10.64 and 10.67 are invalid if (k_{eff}/k) found from them is less than unity. If (k_{eff}/k) is less than one, then the process is one of pure conduction in the fluid and $k_{eff} = k$ should be used.

Example 10.16. A sphere of 0.15 *m* diameter stores a brine at −5°C and is insulated by enclosing it in another sphere of 0.2 *m* diameter and the intervening space contains air at 1 bar. The outside sphere is at 25°C. Estimate the convection heat transfer rate.

Solution.

Data:

$D_i := 0.15$ m $D_o := 0.2$ m $b := 0.025$ m $T_i := -5°C$ $T_o := 25°C$ $g := 9.81\ m/s^2$

We need properties of air at average temperature $T_f = (25 - 5)/2$

$$T_f := 10°C \qquad \textit{(average temperature)}$$

Properties of air at 10°C:

$\nu := 14.19 \times 10^{-6}\ m^2/s$ $k := 0.02487$ W/(mK) $Pr := 0.716$ $\beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.534 \times 10^{-3}$ 1/K

Remember that here, the gap between spheres 'b' is the characteristic dimension.

Then, $$GR_b := \frac{g \cdot \beta \cdot (T_o - T_i) \cdot b^3}{\nu^2}$$

i.e. $$Gr_b = 8.07 \times 10^4 \qquad \textit{(Grashoff number)}$$

Therefore, $$Ra_b := Gr_b \cdot Pr$$

i.e. $$Ra_b = 5.778 \times 10^4 \qquad \textit{(Rayleigh number)}$$

Now,

$$\frac{k_{eff}}{k} = 0.74 \cdot \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} \cdot Ra_{cs}^{\frac{1}{4}} \qquad (10 < Ra_{cs} < 10^6 \ldots(10.67))$$

where $$Ra_{cs} := \frac{b \cdot Ra_b}{D_o^4 \cdot D_i^4 \cdot \left[\frac{1}{D_i^{\frac{7}{5}}} + \frac{1}{D_o^{\frac{7}{5}}}\right]^5} \qquad \ldots(10.68)$$

i.e. $$Ra_{cs} = 235.649$$

Therefore, from Eq. 10.67 we get:

$$\frac{k_{eff}}{k} = 2.38$$

i.e. $$k_{eff} := 2.38 \cdot k$$

i.e. $$k_{eff} = 0.059\ W/(mK) \qquad \textit{(effective thermal conductivity)}$$

Therefore, rate of heat loss:

$$Q := \pi \cdot k_{eff} \cdot \left(\frac{D_i \cdot D_o}{b}\right) \cdot (T_i - T_o) \qquad \ldots(10.66)$$

i.e. $$Q = -6.694\ \mathbf{W.}$$

Note: Negative sign indicates that heat flow is from outside to inside.

Example 10.17. A long tube of 0.1 *m* OD is maintained at 150°C. It is surrounded by a cylindrical radiation shield, located concentrically, such that the air gap between the two cylinders is 10 mm. The shield is at a temperature of 30°C. Estimate the convection heat transfer rate per metre length.

Solution.

Data:

$D_i := 0.1$ m $D_o := 0.12$ m $b := 0.01$ m $T_i := 150°C$ $T_o := 30°C$ $g := 9.81\ m/s^2$

We need properties of air at average temperature $T_f = (150 + 30)/2$

$$T_f := 90°C \qquad \textit{(average temperature)}$$

Properties of air at 90°C:

$\nu := 21.96 \times 10^{-6}$ m²/s $\quad k := 0.03059$ W/(mK) $\quad Pr := 0.705 \quad \beta := \frac{1}{T_f + 273}$ i.e. $\beta = 2.755 \times 10^{-3}$ 1/K

Remember that here, the gap between cylinders '*b*' is the characteristic dimension.

Then, $$Gr_b := \frac{g \cdot \beta \cdot (T_i - T_o) \cdot b^3}{\nu^2}$$

i.e. $Gr_b = 6.725 \times 10^3$ *(Grashoff number)*

Therefore, $Ra_b := Gr_b \cdot Pr$

i.e. $Ra_b = 4.741 \times 10^3$ *(Rayleigh number)*

Now,

$$\frac{k_{eff}}{k} = 0.386 \cdot \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} \cdot Ra_{cc}^{\frac{1}{4}} \qquad (100 < Ra_{cc} < 10^7 \text{...(10.64)})$$

where,

$$Ra_{cc} := \frac{\left(\ln\left(\frac{D_o}{D_i}\right)\right)^4 \cdot Ra_b}{b^3 \cdot \left[\frac{1}{D_i^{\frac{3}{5}}} + \frac{1}{D_o^{\frac{3}{5}}}\right]^5} \qquad \text{...(10.65)}$$

i.e. $Ra_{cc} = 213.597$

Using this value of Ra_{cc} in Eq. 10.64 we get:

$$\frac{k_{eff}}{k} = 1.209$$

i.e. $k_{eff} := 1.209 \cdot k$

i.e. $k_{eff} = 0.037$ W/(mK) *(effective thermal conductivity)*

Therefore, rate of heat loss per meter length:

$$\frac{Q}{L} = \frac{2 \cdot \pi \cdot k_{eff} \cdot (T_i - T_o)}{\ln\left(\frac{D_o}{D_i}\right)} \qquad \text{...(10.63)}$$

i.e. $\frac{Q}{L} = 152.943$ **W/m.**

10.5.14 Natural Convection in Turbine Rotors, Rotating Cylinders, Disks and Spheres

Thermal analysis of shafting, flywheels, turbine blades, and other machine elements is of practical importance, and this involves natural convection heat transfer from a rotating body to surrounding ambient.

Cooling of turbine blades:

Blade is cooled by drilling a blind hole from the root till near the tip of the blade and the coolant circulates through this hole by centrifugal acceleration $r_m.\omega^2$ where r_m is the mean radius of the blade measured from shaft centre and ω is the angular velocity of the blade.

So, now, in Grashoff number, acceleration due to gravity term is replaced by centrifugal acceleration. Therefore,

$$Gr_L = \frac{(r_m \cdot \omega^2) \cdot \beta \cdot \Delta T \cdot L^3}{\nu^2}$$

where L is the length of cooling passage.

In practice, Gr is always > 10^{12} and we use the following equation to find the heat transfer coeff. in fully turbulent flow:

$$Nu_a = \frac{h_a \cdot L}{k} = 0.0246 \cdot \left[\frac{Pr^{1.17} \cdot Gr_L}{1 + 0.495 \cdot Pr^{\frac{2}{3}}} \right]^{0.4} \qquad ...(10.69)$$

Once average heat transfer coefficient is calculated, if d and L are the diameter and length of the hole respectively, total heat transferred is calculated by applying the Newton's law of cooling:

$$Q = h_a \cdot (\pi \cdot d \cdot L) \cdot (T_s - T_a) \qquad ...(10.70)$$

where T_s is the surface temperature of the hole and T_a is the coolant temperature.

Rotating cylinders:

Here, we define a peripheral-speed Reynolds number:

$$Re_\omega = \frac{\pi \cdot D^2 \cdot \omega}{\nu} \qquad ...(10.71)$$

At speeds greater than critical, ($Re_\omega > 8000$ in air), following correlation is used for average Nusselt number in natural convection from a rotating, horizontal cylinder, in air:

$$Nu_D = \frac{h_c \cdot D}{k} = 0.11 \cdot (0.5 \cdot Re_\omega^2 + Gr_D \cdot Pr)^{0.35} \qquad ...(10.72)$$

Rotating disk:

At rotational Reynolds number $\omega . D^2 / \nu$ below about 10^6, boundary layer on the disk is laminar.

For laminar regime, average Nu for a disk rotating in air:

$$Nu_D = \frac{h_a \cdot D}{k} = 0.36 \cdot \left(\frac{\omega \cdot D^2}{\nu} \right)^{\frac{1}{2}} \qquad \left(\text{for } \omega . D^2/\nu < 10^6 ...(10.73)\right)$$

For turbulent regime, local value of Nu at a radius r is given approximately by:

$$Nu_r = \frac{h_c \cdot r}{k} = 0.0195 \cdot \left(\frac{\omega \cdot r^2}{\nu} \right)^{0.8} \qquad ...(10.74)$$

If there is laminar flow between $r = 0$ and $r = r_c$, and turbulent flow between $r = r_c$ and $r = r_0$, average value of Nusselt number is given by:

$$Nu_r = \frac{h_c \cdot r_o}{k} = 0.36 \cdot \left(\frac{\omega \cdot r_o^2}{\nu} \right)^{\frac{1}{2}} \cdot \left(\frac{r_c}{r_o} \right)^2 + 0.015 \cdot \left(\frac{\omega \cdot r_o^2}{\nu} \right)^{0.8} \cdot \left[1 - \left(\frac{r_c}{r_o} \right)^{2.6} \right] \qquad \left(\text{for } r_c < r_o ...(10.75)\right)$$

Rotating sphere:

For $Pr > 0.7$, in laminar flow regime, (i.e. $Re_\omega = \omega . D^2/\nu < 5 \times 10^4$), average Nusselt number is given by:

$$Nu_D = 0.43 \cdot Re_\omega^{0.5} \cdot Pr^{0.4} \qquad \left(Re_\omega < 5 \times 10^4 ...(10.76)\right)$$

And,

$$Nu_D = 0.066 \cdot Re_\omega^{0.67} \cdot Pr^{0.4} \qquad \left(5 \times 10^4 < Re_\omega < 7 \times 10^5 ...(10.77)\right)$$

Example 10.18. A turbine blade is cooled by free convection with water as coolant. The cooling passage is 8 mm in diameter and 8 cm long. The blade velocity at a mean radius of 25 cm is 240 m/s. The hole surface temperature is at 230°C and cooling water temperature is 50°C. Find the average heat transfer coefficient and the rate of heat loss.

Solution.

Data:

$D := 0.008$ m $\quad L := 0.08$ m $\quad r_m := 0.25$ m $\quad V := 240$ m/s $\quad T_s := 230$°C $\quad T_a := 50$°C $\quad g := 9.81$ m/s^2

We need properties of water at average temperature $T_f = (230 + 50)/2$

$T_f := 140$°C ...*average temperature*

Properties of water at 140°C:

$\nu := 0.2118 \times 10^{-6}$ m^2/s $\quad k := 0.6845$ W/(mK) $\quad Pr := 1.23 \quad \beta := 0.966 \times 10^{-3}$ 1/K

Here, the length of hole 'L' is the characteristic dimension.

Then,

$$Gr_L = \frac{(r_m \cdot \omega^2)\cdot \beta \cdot \Delta T \cdot L^3}{\nu^2} \qquad \textit{(Grashoff number)}$$

In the above, $(r_m \cdot \omega^2)$ is the centrifugal acceleration of water at the mean radius, r_m.

Now, $V = r_m \cdot \omega$

i.e. $\omega = \dfrac{V}{r_m}$

And, $r_m \cdot \omega^2 = \dfrac{V^2}{r_m} = 2.304 \times 10^5 \text{ m/s}^2$ *(centrifugal acceleration of water.)*

Then, we get

$$Gr_L := \frac{(2.304 \times 10^5)\cdot \beta \cdot (T_s - T_a)\cdot L^3}{\nu^2} \qquad \textit{(Grashoff number)}$$

i.e. $Gr_L = 4.572 \times 10^{14}$ *(Grashoff number)*

Heat transfer coefficient

We use:

$$Nu_a = \frac{h_a \cdot L}{k} = 0.0246 \cdot \left[\frac{Pr^{1.17}\cdot Gr_L}{1 + 0.495 \cdot Pr^{\frac{2}{3}}}\right]^{0.4} \qquad \text{...(10.69)}$$

i.e. $Nu_a := 1.655 \times 10^4$

i.e. $h_a := \dfrac{Nu_a \cdot k}{L}$ W/(m²K) *(heat transfer coefficient)*

i.e. $h_a = 1.416 \times 10^5$ W/(m²K) *(heat transfer coefficient)*

Heat transfer:

$$Q := h_a \cdot (\pi \cdot D \cdot L)\cdot (T_s - T_a) \qquad \text{...(10.70)}$$

i.e. $Q = 5.125 \times 10^4$ W

Example 10.19. A 15 cm diameter steel shaft whose surface is at 120°C is allowed to cool while rotating about its own horizontal axis at 3 r.p.m. in an environment of air at 20°C. Find the initial rate of heat loss.

Solution.

Data:

$D := 0.15$ m $\quad L := 1$ m $\quad N := 3$ r.p.m $\quad T_s := 120°C \quad T_a := 20°C \quad g := 9.81 \text{ m/s}^2$

We need properties of air of average temperature $T_f = (120 + 20)/2$

$Tf := 70°C$ *(average temperature)*

Properties of air at 70°C:

$\nu := 19.9 \times 10^{-6} \text{ m}^2/\text{s} \quad k := 0.02922$ W/(mK) $\quad Pr := 0.707 \quad \beta := \dfrac{1}{T_f + 273}$ 1/K i.e. $\beta = 2.915 \times 10^{-3}$ 1/K

Now, rotation speed of the shaft is:

$$\omega := \frac{2\cdot \pi \cdot N}{60} \text{ rad/s}$$

i.e. $\omega = 0.314$ rad/s

Here, we have the peripheral—speed Reynolds number

$$Re_\omega := \frac{\pi \cdot D^2 \cdot \omega}{\nu} \qquad \text{...(10.71)}$$

i.e. $Re_\omega = 1.116 \times 10^3$

And,

$$Gr_D := \frac{g \cdot \beta \cdot (T_s - T_a)\cdot D^3}{\nu^2} \qquad \textit{(Grashoff number)}$$

i.e. $Gr_D = 2.437 \times 10^7$ *(Grashoff number)*

and, $Ra := Gr_D \cdot Pr$ *(Rayleigh number)* i.e. $Ra = 1.723 \times 10^7$ *(Rayleigh number)*

Then, from Eq. 10.72:

$$Nu_D = \frac{h_c \cdot D}{k} = 0.11 \cdot (0.5 \cdot Re_\omega^2 + Gr_D \cdot Pr)^{0.35} \quad ...(10.72)$$

i.e. $Nu_D := 37.796$ *(Nusselt number)*

And, $h_c := \frac{Nu_D \cdot k}{D}$ W/(m²K) *(heat transfer coefficient)*

i.e. $h_c = 7.363$ W/(m²K) *(heat transfer coefficient)*

Initial rate of heat loss:

$$Q := h_c \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_a) \text{ W/m}$$

i.e. $Q = 346.957$ W/m.

Example 10.20. A 20 cm diameter disk, being ground at 3000 r.p.m. has its surface at 70°C. Surrounding air is at 30°C. Find the value of convection coefficient

Solution.

Data:

$D := 0.2$ m $\quad N := 3000$ r.p.m $\quad T_s := 70°C \quad T_a := 30°C \quad g := 9.81$ m/s²

We need properties of air at average temperature $T_f = (70 + 30)/2$

$T_f := 50°C$ *(average temperature)*

Properties of air at 50°C:

$\nu := 17.92 \times 10^{-6}$ m²/s $\quad k := 0.02781$ W/(mK) $\quad Pr := 0.709 \quad \beta := \frac{1}{T_f + 273}$ 1/K i.e. $\beta = 3.096 \times 10^{-3}$ 1/K

Now, rotational speed:

$$\omega := \frac{2 \cdot \pi \cdot N}{60}$$

i.e. $\omega = 314.159$ rad/s

Therefore, $\frac{\omega \cdot D^2}{\nu} = 7.012 \times 10^5$ **(this is less than 10^6...therefore, laminar.)**

Then, applying Eq. 10.73 we get:

$$Nu_D = \frac{h_a \cdot D}{k} = 0.36 \cdot \left(\frac{\omega \cdot D^2}{\nu}\right)^{\frac{1}{2}} \quad \text{(for } \omega.D^2/\nu < 10^6 ...(10.73))$$

i.e. $Nu_D := 301.466$ *(Nusselt number)*

And, $h_a := \frac{Nu_D \cdot k}{D}$ W/(m²K) *(heat transfer coefficient)*

i.e. $h_a = 41.919$ **W/(m²K)** ***(heat transfer coefficient)***

Example 10.21. A sphere, 0.1 *m* in diameter is rotating at 30 r.p.m. in a large container of Carbon dioxide at atmospheric pressure. The sphere is at 180°C and the CO_2 is at 20°C. Estimate the rate of heat transfer.

Solution.

Data:

$D := 0.1$ m $\quad N := 30$ r.p.m $\quad T_s := 180°C \quad T_a := 20°C \quad g := 9.81$ m/s²

We need properties of CO_2 at average temperature $T_f = (180 + 20)/2$

$T_f := 100°C$ *(average temperature)*

Properties of CO_2 at 100°C:

$\nu := 12.6 \times 10^{-6}$ m²/s $\quad k := 0.02279$ W/(mK) $\quad Pr := 0.733 \quad \beta := \frac{1}{T_f + 273}$ 1/k i.e. $\beta = 2.681 \times 10^{-3}$ 1/K

Now, rotational speed:

$$\omega := \frac{2 \cdot \pi \cdot N}{60}$$

i.e. $\omega = 3.142$ rad/s

And, $Re_\omega := \frac{\omega \cdot D^2}{\nu}$

i.e. $Re_{\omega} = 2.493 \times 10^3$ *(this is less than 5×10^4...therefore, laminar.)*

Then, average Nusselt number is given by:

$$Nu_D := 0.43 \cdot Re_{\omega}^{0.5} \cdot Pr^{0.4}$$ *($Re_{\omega} < 5 \times 10^4$...(10.76))*

i.e. $Nu_D = 18.963$ *(Nusselt number)*

Then, $$h := \frac{Nu_D \cdot k}{D} \text{ W/(m}^2\text{K)}$$ *(heat transfer coefficient)*

i.e. $h = 4.322$ W/(m^{2}K) *(heat transfer coefficient)*

Heat transfer rate:

$$Q := h \cdot (\pi \cdot D^2) \cdot (T_s - T_a) \text{ W}$$

i.e. $Q = 21.723$ W.

10.5.15 Natural Convection from Finned Surfaces

'Heat sinks' used in cooling of electronic devices have fins on their surfaces. Heat is transferred to the heat sink from the electronic device by conduction and then the heat is dissipated to the ambient from the fins, mostly by natural convection. Advantage of natural convection cooling is that there is no need to have an external moving part (like a fan or pump) and therefore, there is increased reliability. Of course, there is a limitation to the amount of heat that can be transferred, and if the heat to be dissipated is quite large, forced convection cooling may have to be resorted to.

Fins increase the surface area for heat transfer. If the fins are very close to each other, we will have more area, but the heat transfer coefficient will be low since too close a spacing of the fins impedes the flow of fluid by convection. Instead, if the fins are far apart, total surface area will be less, but heat transfer coefficient will be larger. Therefore, there is an optimum spacing for the fins, which maximizes the heat transfer by natural convection from a given base area of width W and height L.

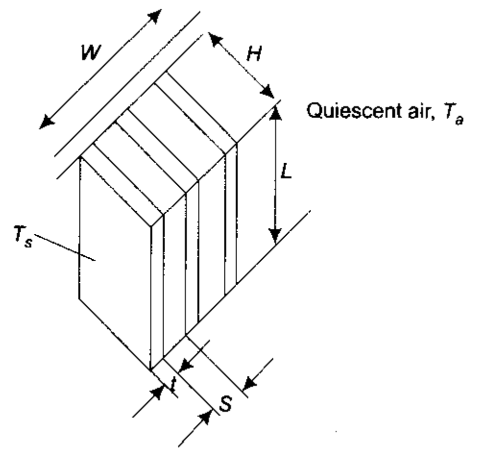

FIGURE 10.6 Free convection from vertical heat sink with fins

Rectangular fins on a vertical surface:
See Fig. 10.6.

For a vertical heat sink with isothermal fins of thickness 't' much smaller than the fin spacing 'S', Bar–Cohen and Rohsenow give the optimum fin spacing as:

$$S_{opt} = 2.714 \cdot \frac{L}{Ra^{\frac{1}{4}}} \quad \text{...(10.78)}$$

where L is the fin length in vertical direction and it is the characteristic dimension to calculate Ra.

Then, heat transfer coefficient for this case of optimum spacing is given by:

$$h = 1.31 \cdot \frac{k}{S_{opt}} \quad \text{...(10.79)}$$

and the rate of heat transfer by natural convection from the fins is determined from;

$$Q = h \cdot (2 \cdot n \cdot L \cdot H) \cdot (T_s - T_a) \quad \text{...(10.80)}$$

where $n = W/(S + t)$ = number of fins and T_s is the surface temperature of fins.

Rectangular fins on a horizontal surface:
See Fig. 10.7.

For rectangular fins on horizontal surfaces, fins facing upwards for $T_s > T_a$ (or facing downward for $T_s < T_a$), Jones and Smith give following correlation:

$$Nu_s = \left[\left(\frac{1500}{Ra_s} \right)^2 + (0.081 \cdot Ra_s^{0.39})^{-2} \right]^{\frac{-1}{2}} \quad \text{...(10.81)}$$

Above equation is valid over the range:

$200 < Ra_s < 6 \times 10^5$, $Pr = 0.71$, $0.026 < H/W < 0.19$, and $0.016 < S/W < 0.20$, with the following definitions:

$$Nu_s = \frac{q \cdot S}{(T_s - T_a) \cdot k} \quad \text{and,} \quad Ra_s = \frac{g \cdot \beta \cdot (T_s - T_a) \cdot S^3}{\nu \cdot \alpha}$$

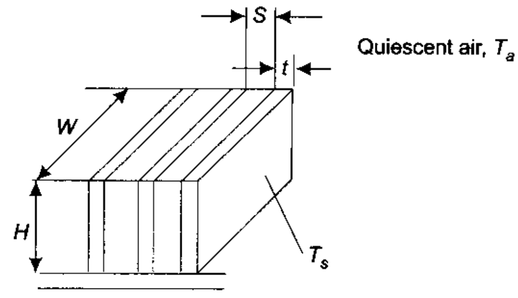

FIGURE 10.7 Rectangular fins on a horizontal surface

Example 10.22. A vertical heat sink, 0.3 *m* wide × 0.15 *m* high, is provided with vertical, rectangular fins of 1 mm thickness. Base and surface temperature of fins is 100°C and the surrounding air is at 20°C. Determine the optimum fin spacing and the rate of heat transfer from the heat sink by natural convection.

Solution.

Data:

$W := 0.3$ m $L := 0.15$ m $H := 0.02$ m

$t := 0.001$ m $T_s := 100°C$ $T_a := 20°C$ $g := 9.81$ m/s^2

We need properties of air at average temperature $T_f = (100 + 20)/2$

$$T_f := 60°C \qquad \textit{(average temperature)}$$

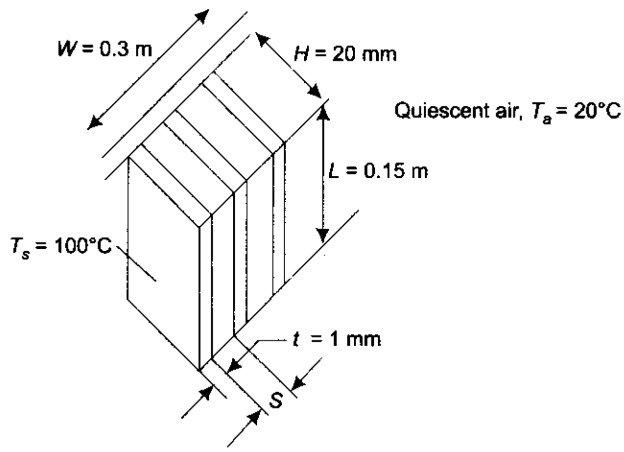

FIGURE Example 10.22 Free convection from vertical heat sink with fins

Properties of air at 60°C:

$\nu := 18.97 \times 10^{-6}$ m^2/s $k := 0.02896$ W/(mK) $Pr := 0.696$ $\beta := \frac{1}{T_f + 273}$ 1/K i.e. $\beta = 3.003 \times 10^{-3}$ 1/K

Now, the characteristic length is the length of fins in vertical direction, i.e. $L = 0.15$ m.

Then,

$$Gr_L := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L^3}{\nu^2} \qquad \textit{(Grashoff number)}$$

i.e. $Gr_L = 2.21 \times 10^7$ *(Grashoff number)*

And, $Ra := Gr_L \cdot Pr$

i.e. $Ra = 1.538 \times 10^7$ *(Rayleigh number)*

Optimum fin spacing:

We use Eq. 10.78:

$$S_{opt} := 2.714 \frac{L}{Ra^{\frac{1}{4}}} \qquad ...(10.78)$$

i.e. $S_{opt} = 6.5 \times 10^{-3}$ m

i.e. $S_{opt} = 6.5$ mm

No. of fins:

$$n := \frac{W}{S_{opt} + t} \qquad (no.\ of\ fins)$$

i.e. $n = 39.998$ (*say* 40)

i.e. $n := 40$ (*no. of fins*)

Heat transfer coefficient:

From Eq. 10.79:

$$h := 1.31 \cdot \frac{k}{S_{opt}} \qquad ...(10.79)$$

i.e. $h = 5.836\ W/(m^2K)$ (*heat transfer coefficient*)

Heat transfer rate:

From Eq. 10.80:

$$Q := h \cdot (2 \cdot n \cdot L \cdot H)\ (T_s - T_a) \qquad ...(10.80)$$

i.e. $Q = 112.056\ W.$

10.6 Comprehensive Correlations from Russian Literature

Following correlations are from the text book by M. Mikheyev.

Free convection from different objects:

Free convection from different objects were investigated with various fluids such as air, hydrogen, carbon dioxide, water, aniline, glycerine, carbon tetrachloride, various oils etc. Objects studied included horizontal and vertical wires, tubes, plates, and spheres of widely different sizes: wires and tubes from 0.015 to 245 mm in diameter, spheres from 30 mm to 16 *m* in diameter, height of plates and tubes ranging from 0.25 to 6 m. Gas pressures were varied from 0.03 to 70 ata.

While generalizing the data, reference dimension was diameter *d* for tubes and spheres, and height *h* for plates. Properties of fluids were taken at film temperature, $T_f = (T_s + T_a)/2$. It is interesting to note that all the data, when plotted with log(*Gr.Pr*) on the *x*-axis and log (*Nu*) on the *y*-axis, fall fairly well on one common curve. So, the general relation is:

$$Nu_f = C(Gr \cdot Pr)^n \qquad ...(10.82)$$

Values of C and *n* for different ranges of $(Gr \cdot Pr)$ are taken from following Table:

$(Gr \cdot Pr)_f$	C	n
$1 \times 10^{-3} - 5 \times 10^2$	1.18	1/8
$5 \times 10^2 - 2 \times 10^7$	0.54	1/4
$2 \times 10^7 - 1 \times 10^{13}$	0.135	1/3

Note that with $Ra = (Gr \cdot Pr) < 1$, $Nu = 0.5$ and remains constant, i.e. $h = 0.5.k/d$ = heat transfer coefficient for very low Rayleigh numbers. (e.g. for very thin wires).

Principal conclusions were: (a) Rayleigh number is the main dimensionless term to determine heat transfer in fee convection (b) Shape of the body is of secondary importance in the process considered.

Eq. 10.82 is applicable to any fluid with $Pr > 0.7$ and for bodies of any shape and size. Same formula may be used to calculate heat transfer from horizontal plates too. Then reference dimension is the smaller side of the plate. Value of *h* determined from Eq. 10.82 must be increased by 30 % if the hot surface is facing upwards, and decreased by 30 % if the heat losing surface faces downward.

For horizontal tubes, especially, following correlation is recommended for free convection with liquids and gases:

$$Nu_a = 0.51 \cdot (Gr \cdot Pr)^{\frac{1}{4}} \cdot \left(\frac{Pr_a}{Pr_w}\right)^{\frac{1}{4}} \qquad ...(10.83)$$

Here, note that fluid properties are determined at free stream temperature T_a and the reference dimension is the tube diameter, *d*.

For air, Eq. 10.83 is given in the following simplified form:

$$Nu_a = 0.47 \cdot Gr_a^{\frac{1}{4}} \qquad ...(10.84)$$

Note that for horizontal tubes, Eq. 10.83 is to be preferred to Eq. 10.82.

Free convection in different enclosures:

Here, the concept of 'equivalent thermal conductivity' is used. It has the advantage that the heat transfer coefficient h need not be determined. In the following correlations, thickness of the enclosure δ and the mean fluid temperature ($= [T_1 + T_2]/2$), are taken as the reference dimension and reference temperature respectively, irrespective of the shape of the enclosure. Passages considered are: horizontal passages, vertical passages, enclosures within concentric cylinders and concentric spheres. Again, it is found that for all these enclosures, data fall well within a single curve. Following are the correlations:

$$\frac{k_{eff}}{k} = 1 \qquad (for\ Ra < 1000...(10.85))$$

$$\frac{k_{eff}}{k} = 0.105 \cdot Ra^{0.3} \qquad (for\ 10^3 < Ra < 10^6...(10.86))$$

and,

$$\frac{k_{eff}}{k} = 0.4 \cdot Ra^{0.2} \qquad (for\ 10^6 < Ra < 10^{10}...(10.87))$$

In approximate calculations, Eqs. 10.86 and 10.87 may be replaced by the following single eqn. for the entire range of $Ra > 1000$:

$$\frac{k_{eff}}{k} = 0.18 \cdot Ra^{0.25} \qquad (for\ Ra > 1000...(10.88))$$

If k_{eff}/k works out to be less than one, it means that $Ra < 1000$, and we should take $k_{eff} = k$.

Example 10.23. Work out Example 10.11 with formula from Russian literature: A sphere of 25 mm diameter, with its surface temperature at 100°C, is kept in still air at a temperature of 20°C. Determine the rate of convective heat loss.

Solution.

Data:

$D := 25 \times 10^{-3}$ m $\quad T_s := 100°C \quad T_a := 20°C \quad g := 9.81\ m/s^2$

Properties of air at film temperature of (100 + 20)/2 = 60°C are:

$T_f := 60°C \quad \nu := 18.97 \times 10^{-6}\ m^2/s \quad k := 0.02896\ W/(mK) \quad Pr := 0.696 \quad \beta := \frac{1}{T_f + 273}\ 1/K$

i.e. $\beta = 3.003 \times 10^{-3}\ 1/K$

Diameter D is the characteristic dimension to calculate Ra.

$A := \pi \cdot D^2\ m^2$ *(surface area of sphere)*

i.e. $A = 1.963 \times 10^{-3}\ m^2$ *(surface area)*

And, $Gr := \frac{D^3 \cdot g \cdot \beta \cdot (T_s - T_a)}{\nu^2}$

i.e. $G_r = 1.023 \times 10^5$ *(Grashoff number)*

and, $Ra := Gr \cdot Pr$

i.e. $Ra = 7.122 \times 10^4$ *(Rayleigh number)*

Now, use Eq. 10.82:

$$Nu_f = C \cdot (Gr \cdot Pr)^n \qquad ...(10.82)$$

From the Table, for $Ra = 7.122 \times 10^4$, we get:

$$C = 0.54 \quad \text{and,} \quad n = \frac{1}{4}$$

Therefore, $Nu_f := C \cdot Ra^n$ *(Nusselt number)*

i.e. $Nu_f = 8.822$ *(Nusselt number)*

And, $h := \frac{Nu_f \cdot k}{D}\ W/(m^2K)$ *(heat transfer coefficient)*

i.e. $h = 10.219\ W/(m^2K)$ *(heat transfer coefficient)*

Compare this value of h with $h = 10.454$ W/(m^{2}K) obtained earlier.

Therefore, rate of heat loss from sphere:

$$Q := h \cdot A \cdot (T_s - T_a) \text{ W}$$

i.e. $$Q = 1.605 \textbf{ W}.$$

Compare this value of Q with $Q = 1.642$ W, obtained earlier.

Example 10.24. Work out Example 10.16 with formula from Russian literature: A sphere of 0.15 m diameter stores a brine at −5°C and is insulated by enclosing it in another sphere of 0.2 m diameter and the intervening space contains air at 1 bar. The outside sphere is at 25°C. Estimate the convection heat transfer rate.

Solution.

Data:

$D_i := 0.15$ m $\quad D_o := 0.2$ m $\quad b := 0.025$ m $\quad T_i := -5$°C $\quad T_o := 25$°C $\quad g := 9.81$ m/s^2

We need properties of air at average temperature $T_f = (25 - 5)/2$

$$T_f := 10°\text{C}$$ *...average temperature*

Properties of air at 10°C:

$\nu := 14.19 \times 10^{-6}$ m^2/s $\quad$ k := 0.02487 W/(mK) $\quad Pr := 0.716 \quad \beta := \dfrac{1}{T_f + 273}$ i.e. $\beta = 3.534 \times 10^{-3}$ 1/K

Remember that here, the gap between spheres 'b' is the characteristic dimension.

Then, $$Gr_b := \frac{g \cdot \beta \cdot (T_o - T_i) \cdot b^3}{v^2}$$

i.e. $$Gr_b = 8.07 \times 10^4$$ (Grashoff number)

Therefore, $$Ra_b := Gr_b \cdot Pr$$

i.e. $$Ra_b = 5.778 \times 10^4$$ *(Rayleigh number)*

For this value of Ra, appropriate equation is Eq. 10.86 viz.

$$\frac{k_{eff}}{k} = 0.105 \cdot Ra_b^{0.3}$$ *(for $10^3 < Ra_b < 10^6$...(10.86))*

i.e. $$\frac{k_{eff}}{k} = 2.817$$

i.e. $$k_{eff} := 2.817 \cdot k$$

i.e. $$k_{eff} = 0.07 \text{ W/(mK)}$$ *(effective thermal conductivity)*

Compare this value with $k_{eff} = 0.059$ W/(mK), obtained earlier.

Therefore, rate of heat loss:

$$Q := \pi \cdot k_{eff} \left(\frac{D_i \cdot D_o}{b}\right) \cdot (T_i - T_o)$$ ....(10.66)

i.e. $$Q = -7.923 \text{ W}$$

Note: Negative sign indicates that heat flow is from outside to inside.

Compare this value of Q with $Q = -6.694$ W, obtained earlier.

10.7 Combined Natural and Forced Convection

In many practical situations, natural and forced convection may occur together. At high velocities forced convection may be predominant, but at low velocities effect of natural convection also must be included. Further, natural and forced convection may occur in the same direction or they may act in opposite directions. We have the following criteria to determine if the combined free and forced convection is to be considered.

$Gr_L/(Re_L^2) << 0.1$ *(forced convection regime (negligible free convection)*

$Gr_L/(Re_L^2) >> 10$ *(free convection regime (negligible forced convection)*

$0.1 < Gr_L/(Re_L^2) \approx 10$ *(mixed convection regime (both free and forced convection are important)*

In the mixed convection regime, following equation is used to calculate the Nusselt number:

$$Nu^m = Nu_{forced}^m \pm Nu_{free}^m$$ .....(10.89)

where first and second terms on RHS are Nusselt numbes for forced and free convection respectively. A value of $m = 3$ is generally recommended. Positive or negative sign is taken if the free convection flow occurs in the same or opposite direction to that of forced convection.

For the specific case of **mixed convection for internal flow through a horizontal pipe**, we have the following correlations for average Nusselt number:

For laminar flow ($Re_D \le 2000$): (correlation due to Brom and Gauwin):

$$Nu_D = 1.75 \cdot \left(\frac{\mu_b}{\mu_s}\right)^{0.14} \cdot \left[Gz + 0.012 \cdot \left(Gz \cdot Gr_D^{\frac{1}{3}}\right)^{\frac{4}{3}}\right]^{\frac{1}{3}} \qquad ...(10.90)$$

where μ_b and μ_s are viscosities of the fluid at the bulk mean temperature and surface temperature respectively, and Gz is the Graetz number, given by:

$$Gz = Re_D \cdot Pr \cdot \left(\frac{D}{L}\right) = \text{Graetz number.} \qquad ...(10.91)$$

For turbulent flow : (correlation due to Metais and Eckert):

$$Nu_D = 4.69 \cdot Re_D^{0.27} \cdot Pr^{0.21} \cdot Gr_D^{0.07} \left(\frac{D}{L}\right)^{0.36} \qquad ...(10.92)$$

Example 10.25. An un-insulated pipe of 50 mm OD, with a surface temperature of 50°C, runs through a plant room. An exhaust fan creates a mild flow of air upwards across the pipe, with a velocity of 0.2 m/s. If the ambient temperature is 30°C, calculate the rate of heat loss by combined free and forced convection.

Solution.

Data:

$D := 0.05$ m $\quad L := 1$ m $\quad V := 0.2$ m/s $\quad T_s := 50°C \quad T_a := 30°C \quad g := 9.81$ m/s^2

We need properties of air at average temperature $T_f = (50 + 30)/2$

$T_f := 40°C$ *(average temperature)*

Properties of air at 40°C:

$\nu := 16.96 \times 10^{-6}$ m^2/s $\quad k := 0.0271$ W/(mK) $\quad Pr := 0.71 \quad \beta := \frac{1}{T_f + 273}$ i.e. $\beta = 3.195 \times 10^{-3}$ 1/K

Remember that here, the diameter of pipe, D, is the characteristic dimension.

Then, $Gr_D := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot D^3}{\nu^2}$

i.e. $Gr_D = 2.724 \times 10^5$ *(Grashoff number)*

Therefore, $Ra_D := Gr_D \cdot Pr$

i.e. $Ra_D = 1.934 \times 10^5$ *(Rayleigh number)*

Now, Reynolds number is given by:

$Re := \frac{D \cdot V}{\nu}$

i.e. $Re = 589.623$ *(Reynold number)*

Therefore,

$\frac{Gr_D}{Re^2} = 0.784$ *(this value is nearly equal to one. Therefore, flow is in mixed convection regime. i.e. both free and forced convection must be considered.)*

Free convection Nusselt number:
From Eq. 10.43:

$$Nu_{free} = \left[0.60 + 0.387 \cdot \left[\frac{Ra_D}{\left[1 + \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}}}\right]^{\frac{1}{6}}\right]^2 \qquad (10^{-5} < Ra_D < 10^{12} ...(10.43))$$

i.e. $Nu_{free} = 9.261$ *(Free convection Nusselt number)*

Forced convection Nusselt number:
Using Churchill and Burnstein correlation:

$$Nu_{cyl} = \frac{h \cdot D}{k} = 0.3 + \frac{0.62 \cdot Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \cdot \left[1+\left(\frac{Re}{28200}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}} \qquad ...(9.90)$$

for $100 < Re < 10^7$ and $Re \cdot Pr > 0.2$

From Eq. 9.90 we get:

$Nu_{forced} = 12.927$ *(Forced convection Nusselt number)*

We use Eq. 10.89 to determine the Nusselt number for mixed convection. Also, we use the '+' sign, since both the free convection from hot pipe and forced convection with air flowing from bottom of pipe upwards, are additive. We use the value of exponent as 3.

i.e. $Nu_{mixed}^3 = Nu_{free}^3 + Nu_{forced}^3$ ...(10.89)

i.e. $Nu_{mixed}^3 = 794.293 + 2160 = 2954$

Therefore, $Nu_{mixed} := 14.348$ *(mixed Nusselt number)*

Therefore, combined heat transfer coefficient:

$$h := \frac{Nu_{mixed}\, k}{D}$$

i.e. $h = 7.777\ W/(m^2K)$ *(combined heat transfer coefficient)*

Heat loss per meter length of pipe:

$$Q = h \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_a)\ W/m$$

i.e. $Q = 19.545\ W/m.$

Example 10.26 Air at 1 atm. and 20°C is forced through a 15 mm diameter tube at an average velocity of 20 cm/s. Tube wall is maintained at a temperature of 100°C. The tube is 1 m long. Calculate the rate of heat transfer.

Solution.

Data:

$D := 0.015\ m \quad L := 1.0\ m \quad V := 0.2\ m/s \quad T_s := 100°C \quad T_a := 20°C \quad g := 9.81\ m/s^2$

Properties of air at bulk mean temperature of 20°C:

$\nu := 15.06 \times 10^{-6}\ m^2/s \quad \mu_b := 18.14 \times 10^{-6}\ kg/(ms) \quad k := 0.02593\ W/(mK) \quad Pr := 0.703 \quad \beta := \frac{1}{T_a + 273}$

i.e. $\beta = 3.413 \times 10^{-3}\ 1/K \quad \mu_s := 21.87 \times 10^{-6}\ kg/(ms)$ *(dynamic viscosity of air at surface temperature of* 100°C.)

Reynolds number:

$$Re_D := \frac{V \cdot D}{\nu} \qquad \textit{(Reynolds number)}$$

i.e. $Re_D = 199.203$ *(Reynolds number)*

Grashoff number:

$$Gr_D := \frac{g \cdot \beta \cdot (T_s - T_a) \cdot D^3}{\nu^2} \qquad \textit{(Grashoff number)}$$

i.e. $Gr_D = 3.986 \times 10^4$ *(Grashoff number)*

Therefore,

$$\frac{Gr_D}{Re_D^2} = 1.004$$

Therefore, this is a case of mixed convection i.e. both free and forced convection are to be considered. And, Eq. 10.90 for average Nusselt number is applicable. i.e.

$$Nu_D = 1.75 \left(\frac{\mu_b}{\mu_s}\right)^{0.14} \cdot \left[Gz + 0.012 \cdot \left(Gz \cdot Gr_D^{\frac{1}{3}}\right)^{\frac{4}{3}}\right]^{\frac{1}{3}} \qquad ...(10.90)$$

Now, Graetz number is:

$$Gz := Re_D \cdot Pr \cdot \frac{D}{L} \qquad \textit{(Graetz number)}$$

i.e. $Gz = 2.101$ *(Graetz number)*

Therefore,

$$Nu_D := 1.75 \cdot \left(\frac{\mu_b}{\mu_s}\right)^{0.14} \cdot \left[Gz + 0.012 \cdot \left(Gz \cdot Gr_D^{\frac{1}{3}}\right)^{\frac{4}{3}}\right]^{\frac{1}{3}}$$

i.e. $Nu_D = 3.041$ ...*Average Nusselt number*

Then, $h := \dfrac{Nu_D \cdot k}{D}$ W/(m²K) ...*heat transfer coefficient*

i.e. $h = 5.258$ W/(m²K) ...*heat transfer coefficient*

Heat transfer rate/m length:

$$Q := h \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_a) \text{ W/m}$$

i.e. $Q = 19.821$ **W/m.**

10.7 Summary of Basic Equations for Natural Convection

Important correlations are summarized below:

Geometry	*Correlation*
Heated, vertical plate: Integral method:	Temperature distribution: $\dfrac{T - T_a}{T_s - T_a} = \left(1 - \dfrac{y}{\delta}\right)^2$ Velocity distribution: $\dfrac{u}{u_x} = \dfrac{y}{\delta} \cdot \left(1 - \dfrac{y}{\delta}\right)^2$ Maximum velocity: $u_{max} = \dfrac{4}{27} \cdot u_x$ at $y = \delta/3$. Mean velocity: $u_m = \dfrac{1}{12} \cdot u_x = \dfrac{27}{48} \cdot u_{max}$ Velocity function: $u_x = 5.17 \cdot \nu \cdot (Pr + 0.952)^{-0.5} \cdot \left[\dfrac{\beta \cdot g \cdot (T_s - T_a)}{\nu^2}\right]^{0.5} \cdot x^{0.5}$ Boundary layer thickness: $\dfrac{\delta}{x} = \dfrac{3.93 \cdot (0.952 + Pr)^{0.25}}{Gr_x^{0.25} \cdot Pr^{0.5}}$ Total mass flow through the boundary: $m_{total} = 1.7 \rho \cdot \nu \left[\dfrac{Gr_L}{Pr^2 \cdot (Pr + 0.952)}\right]^{0.25}$ Average Nusselt number for laminar flow: $Nu_{avg} = \dfrac{4}{3} \cdot Nu_L = \dfrac{0.0667 \cdot Pr^{0.5} \cdot Gr_L^{0.25}}{(0.952 + Pr)^{0.25}}$
	Average Nusselt number for turbulent flow: $Nu_{avg} = \dfrac{h_{avg} \cdot L}{k} = 0.0246 \left[\dfrac{Pr^{1.17} \cdot Gr_L}{1 + 0.495 \cdot Pr^{\frac{2}{3}}}\right]^{0.4}$ ...for turbulent flow.

Contd.

Contd.

Empirical relations:	
Vertical plate, T_s = constant For air and other gases	Height L is the characteristic length. $Nu = 0.59 \cdot Ra^{\frac{1}{4}}$ ...$10^4 < Ra < 10^9$ $Nu = 0.13 \cdot Ra^{\frac{1}{3}}$ ...$10^9 < Ra < 10^{12}$
For all Prandl numbers:b 0< Pr < ∞:	$Nu = 0.68 + \dfrac{0.670 \cdot Ra^{\frac{1}{4}}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$ ...$0 < Ra < 10^9$
0.6 < Pr < ∞: (For high Prandtl No. fluids)	$Nu = \dfrac{0.15 \cdot Ra^{\frac{1}{3}}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{27}}}$ ...$Ra > 10^9$
0 < Pr < 0.6: (Entire range of Ra) (For low Prandtl No. fluids i.e. liquid metals)	$Nu = \left[0.825 + \dfrac{0.387 \cdot Ra^{\frac{1}{6}}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2$ ...$Ra > 10^9$
Inclined plate, inclined at an angle θ to the vertical T_s = constant	Inclined height L is the characteristic length. Use vertical plate equations as a first approximation. Replace g by $g \cdot \cos(\theta)$.
Vertical cylinder	Height L is the characteristic length. Vertical cylinder can be treated as vertical plate, if the following relation is satisfied: $\dfrac{D}{L} \geq \dfrac{34}{Ra^{\frac{1}{4}}}$
Vertical plate, q_s = constant	Eqs. 10.25 and 10.26 are still valid, with the modification that constant 0.492 is changed to 0.437. **Alternatively:** A modified Grashoff number is defined: $Gr' = Gr \cdot Nu_x = \dfrac{g \cdot \beta \cdot q_s \cdot x^4}{k \cdot v^2}$
	And following two relations for local Nusselt no.: $Nu_x = 0.60\ (Gr' \cdot Pr)^{0.2}$ ...$10^5 < Gr'_x < 10^{11}$ Nux = $0.17\ (Gr' \cdot Pr)^{0.25}$ ...$Gr_x > 10^{11}$ For Average Nu: $h = \dfrac{5}{4} \cdot h_L$ ...for laminar

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	$h = h_L$ ...for turbulent.
Horizontal plate, T_s = constant	Characteristic Length: $L_c = A/P$ Upper surface of hot plate (or lower surface of cold plate): $Nu = 0.54 \cdot Ra^{\frac{1}{4}}$ ...$10^4 < Ra < 10^7$ $Nu = 0.15 \cdot Ra^{\frac{1}{3}}$ ...$10^7 < Ra < 10^{11}$ Lower surface of hot plate (or upper surface of cold plate): $Nu = 0.27 \cdot Ra^{\frac{1}{4}}$ ...$10^5 < Ra < 10^{11}$
Horizontal plate, q_s= constant	Characteristic Length: $L_c = A/P$ All property values, except β, are evaluated at a temperature, T_e, defined by: $T_e = T_s - 0.25\,(T_s - T_a)$ and, β is evaluated at T_a. Upper surface of hot plate (or lower surface of cold plate): $Nu = 0.13 \cdot Ra^{\frac{1}{3}}$ ...$Ra < 2 \times 10^8$ $Nu = 0.16 \cdot Ra^{\frac{1}{3}}$ ...$2 \times 10^8 < Ra < 10^{11}$ For heated surface facing downward: $Nu = 0.58\ Ra^{0.2}$ ...$10^6 < Ra < 10^{11}$
Horizontal cylinder, T_s= constant	Diameter D is the characteristic length. For air: $Nu = C \cdot Ra^n$ C and n from Table 10.2. For $(0 \le Pr \le \infty)$: $Nu = \left[0.60 + 0.387 \cdot \left[\dfrac{Ra}{\left[1+\left(\dfrac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}}}\right]^{\frac{1}{6}}\right]^2$ ...$10^{-5} < Ra < 10^{12}$
	And, only for laminar range: $Nu = 0.36 + \dfrac{0.518 \cdot Ra^{\frac{1}{4}}}{\left[1+\left(\dfrac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$ ...$10^{-6} < Ra < 10^9$
For thin wires: (D = 0.2 to 1 mm)	$Nu_D = 1.18 \cdot (Ra_D)^{\frac{1}{8}}$ ...$Ra < 500$
From horizontal cylinders to liquid metals:	$Nu_D = 0.53 \cdot (Gr_D \cdot Pr^2)^{\frac{1}{4}}$

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Spheres:	Diameter D is the characteristic length. $Nu = 2 + 0.43 \cdot (Ra)^{\frac{1}{4}}$ ...$1 < Ra < 10^5$, $Pr = 1$ And, for higher range of Ra: $Nu = 2 + 0.50 \cdot (Ra)^{\frac{1}{4}}$ ...$3 \times 10^5 < \text{Ra} < 8 \times 10^8$
Rectangular blocks:	Ch. Lengh: $L = \dfrac{L_H \cdot L_V}{L_H + L_V}$ $Nu_L = 0.55 \cdot (Ra_L)^{\frac{1}{4}}$ ...$10^4 < Ra_L < 10^9$
Short cylinders ($D = H$)	$Nu = 0.775(Ra)^{0.208}$
Simplified equation for air:	Refer to Table 10.3
Free convection in enclosed spaces: **For Horizontal enclosure:** For air: **For liquids (water, silicone oils and mercury):** **For Vertical enclosure:** For Air:	Space between the plates, 'b' is the characteristic dimension. $Gr_b = \dfrac{g \cdot \beta \cdot (T_1 - T_2) \cdot b^3}{\nu^2}$ $Nu = 0.195 \cdot Gr^{\frac{1}{4}}$ ...$10^4 < Gr < 3.7 \times 10^5$ and, and, $Nu = 0.068 \cdot Gr^{\frac{1}{3}}$ ...$3.7 \times 10^5 < \text{Gr} < 10^7$ And, for $Gr < 1700$, we have $Nu = 1$. $Nu = 0.069 \cdot Ra^{\frac{1}{3}}\ Pr^{0.074}$ ...$1.5 \times 10^5 < Ra < 10^9$ For Gr (based on plate spacing 'b') < 1700, we have $Nu = 1$. $\dfrac{k_{eff}}{k} = Nu = \dfrac{0.18 \cdot Gr^{\frac{1}{4}}}{\left(\dfrac{L}{b}\right)^{\frac{1}{9}}}$ ...$2 \times 10^4 < Gr < 2 \times 10^5$ where k_{eff} = effective thermal conductivity. and, $\dfrac{k_{eff}}{k} = Nu = \dfrac{0.065 \cdot Gr^{\frac{1}{4}}}{\left(\dfrac{L}{b}\right)^{\frac{1}{9}}}$ ...$2 \times 10^5 < Gr < 10^7$ **Note** that for above two relations, aspect ratio, $L/b > 3$. If $L/b < 3$, each vertical surface is treated independently.
For fluids with $Pr = 3$ and 30,000 inside vertical enclosure:	$Nu = 1$ ...for $Ra < 1000$ and, $Nu = \dfrac{0.28 \cdot Ra^{\frac{1}{4}}}{\left(\dfrac{L}{b}\right)^{\frac{1}{4}}}$ ...for $1000 < Ra < 10^7$

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Free convection in inclined spaces: (Flat plate solar collectors and double glazed windows)	For ($L/\delta < 12$) and at tilt angles τ less than 70 deg.: $Nu_L = 1 + 1.44\left(1-\frac{1708}{Ra_L\cdot\cos(\tau)}\right)\cdot\left[1-\frac{1708\cdot(\sin(1.8\cdot\tau))^{1.6}}{Ra_L\cdot\cos(\tau)}\right] + \left[\left(\frac{Re_L\cdot\cos(\tau)}{5830}\right)^{\frac{1}{3}}-1\right]$ If the quantity in the first bracket and the last bracket is negative, then it must be set equal to zero. For tilt angles between 70 deg. and 90 deg. Catton recommends that the Nusselt number for a vertical enclosure (τ = 90 deg.) be multiplied by $(\sin\,\tau)^{1/4}$, i.e. $Nu_L(\tau) = Nu_L(\tau = 90)\cdot(\sin(\tau))^{\frac{1}{4}}$
Natural convection inside spherical cavities:	$\frac{D\cdot h_{avg}}{k} = C\cdot(Gr_D\cdot Pr)^n$ For values of C and n, see table in text.
Concentric cylindrical annuli:	'b' is the gap or thickness of the enclosed fluid layer (i.e. $b = [D_o - D_i]/2$). $\frac{Q}{L} = \frac{2\cdot\pi\cdot k_{eff}\,(T_i - T_o)}{\ln\left(\frac{D_o}{D_i}\right)}$ $\frac{k_{eff}}{k} = 0.386\cdot\left(\frac{Pr}{0.861+Pr}\right)^{\frac{1}{4}}\cdot Ra_{cc}^{\frac{1}{4}}$ ...$100 < Ra_{cc} < 10^7$. and, $Ra_{cc} = \frac{\left(\ln\left(\frac{D_o}{D_i}\right)\right)^4\cdot Ra_b}{b^3\cdot\left[\frac{1}{D_i^{\frac{3}{5}}}+\frac{1}{D_o^{\frac{3}{5}}}\right]^5}$
Concentric spherical annuli:	$Q = \pi\cdot k_{eff}\cdot\left(\frac{D_i\cdot D_o}{b}\right)\cdot(T_i - T_o)$ $\frac{k_{eff}}{k} = 0.74\cdot\left(\frac{Pr}{0.861+Pr}\right)^{\frac{1}{4}}\cdot Ra_{cs}^{\frac{1}{4}}$ ...$10^2 < Ra_{cs} < 10^4$ and, $Ra_{cs} = \frac{b\cdot Ra_b}{D_o^4\cdot D_i^4\cdot\left[\frac{1}{D_i^{\frac{7}{5}}}+\frac{1}{D_o^{\frac{7}{5}}}\right]^5}$

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Cooling of turbine blades: (hole diameter D, hole length, L)	$Gr_L = \frac{(r_m \cdot \omega^2) \cdot \beta \cdot \Delta \cdot T \cdot L^3}{\nu^2}$ Mostly, $Gr_L > 10^{12}$, and we use: $Nu_a = \frac{h_a \cdot L}{k} = 0.0246 \cdot \left[\frac{Pr^{1.17} \cdot Gr_L}{1 + 0.495 \cdot Pr^{\frac{2}{3}}}\right]^{0.4}$ Total heat transferred: $Q = h_a \cdot (\pi \cdot d \cdot L) \cdot (T_s - T_a)$
Rotating cylinders:	Peripheral-speed Reynolds number: $Re_\omega = \frac{\pi \cdot D^2 \cdot \omega}{\nu}$ For ($Re_\omega > 8000$ in air): Average Nusselt number: $Nu_D = \frac{h_c \cdot D}{k} = 0.11 \left(0.5 \cdot \text{Re}_\omega^2 + Gr_D \cdot Pr\right)^{0.35}$
Rotating disk:	For laminar regime, average Nu for a disk rotating in air: $Nu_D = \frac{h_a \cdot D}{k} = 0.36 \cdot \left(\frac{\omega \cdot D^2}{\nu}\right)^{\frac{1}{2}}$ ...for $\frac{\omega \cdot D^2}{\nu} < 10^6$ For laminar flow between $r = 0$ and $r = r_c$, and turbulent flow between $r = r_c$ and $r = r_0$, average value of Nusselt number is given by:
	$Nu_r = \frac{h_c \cdot r_o}{k} = 0.36 \cdot \left(\frac{\omega \cdot r_o^2}{\nu}\right)^{\frac{1}{2}} \cdot \left(\frac{r_c}{r_o}\right)^2 + 0.015 \cdot \left(\frac{\omega \cdot r_o^2}{\nu}\right)^{0.8} \cdot \left[1 - \left(\frac{r_c}{r_o}\right)^{2.6}\right]$ ...for $r_c < r_o$
	For turbulent regime, local value of Nu at a radius r is given by: $Nu_r = \frac{h_c \cdot r}{k} = 0.0195 \cdot \left(\frac{\omega \cdot r^2}{\nu}\right)^{0.8}$
Rotating sphere:	For $Pr > 0.7$, in laminar flow regime, (i.e. $Re_\omega = \omega . D^2/\nu < 5 \times 10^4$), average Nusselt number is given by: $Nu_D = 0.43 Re_\omega^{0.5} Pr^{0.4}$ ...$Re_\omega < 5 \times 10^4$ and, $Nu_D = 0.066\ Re_\omega^{0.67} \cdot Pr^{0.4}$ ...$5 \times 10^4 < Re_\omega < 7 \times 10^5$
Rectangular fins on a vertical surface: **See Fig. 10.6.**	Optimum fin spacing: $S_{opt} = 2.714 \frac{L}{Ra^{\frac{1}{4}}}$ where L = fin length in vertical direction and is also the characteristic length. $h = 1.31 \cdot \frac{k}{S_{opt}}$ Rate of heat transfer: $Q = h \cdot (2 \cdot n \cdot L \cdot H) \cdot (T_s - T_a)$ where n = no. of fins

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Rectangular fins on a horizontal surface: See Fig. 10.7	For fins facing upwards for $T_s > T_a$ (or facing downward for $T_s < T_a$): $$Nu_s = \left[\left(\frac{1500}{Ra_s}\right)^2 + (0.081 \cdot Ra_s^{0.39})^{-2}\right]^{-\frac{1}{2}}$$ Above equation is valid over the range: $200 < Ra_s < 6 \times 10^5$, $Pr = 0.71$, $0.026 < H/W < 0.19$, and $0.016 < S/W < 0.20$, with the following definitions: $$Nu_s = \frac{q \cdot S}{(T_s - T_a) \cdot k}$$ and, $$Ra_s = \frac{g \cdot \beta \cdot (T_s - T_a) \cdot S^3}{\nu \cdot \alpha}$$
Combined Natural and Forced convection	$Gr_L/(Re_L^2) << 0.1$ ....forced convection regime (negligible free convection) $Gr_L/(Re_L^2) >> 10$ ....free convection regime (negligible forced convection) $0.1 < Gr_L/(Re_L^2) \approx 10$ ....mixed convection regime (both free and forced convection are important) In the mixed convection regime, following equation is used to calculate the Nusselt number: $Nu^m = Nu_{forced}^m \pm Nu_{free}^m$ A value of $m = 3$ is generally recommended. Positive or negative sign is taken if the free convection flow occurs in the same or opposite direction to that of forced convection.

10.8 Summary

In natural (or, free) convection, fluid flow is caused by density differences as a result of temperature differences. Natural convection is, in fact, a preferred mode of heat transfer in many practical applications, since there is no need for an external fan or pump to cause the flow, and is therefore more economical and reliable. Cooling of electronic devices, transformers, motors, transmission lines, etc. are some of the common examples of applications of natural convection heat transfer.

In this chapter, first, an outline of the method of solution of the relevant conservation equations by the approximate integral method was given. Solutions for the case of natural convection are move difficult as compared to the case of forced convection since in the case of natural convection, the momentum and energy equations are 'mutually coupled' which means that they have to be solved simultaneously. Next, empirical relations for several geometries and situations of practical importance were listed. Several examples have been worked out, demonstrating the use of these correlations.

Questions

1. Explain the circumstances under which natural convection occurs. Differentiate between natural and forced convection.
2. What is the criterion from laminar to turbulent flow in natural convection?
3. What is the physical significance of Grashoff number? Compare it with Reynolds number.
4. Use the principle of dimensional analysis to establish a relationship between Nusselt number, Grashoff number and Prandtl number. [M.U.]
5. Why is the analytical solution of free convection problems more involved as compared to forced convection problems?
6. State two important applications of heat transfer in an enclosure. What is meant by 'aspect ratio' of an enclosure?

7. What is a 'heat sink'? Why are fins provided in a heat sink?
8. Explain why there is an 'optimum' spacing between fins in a heat sink.
9. What is the criterion to decide if the natural convection is negligible or not, in forced convection heat transfer?
10. How is the Nusselt number calculated in 'mixed convection' regime?

Problems

1. A hot plate 35 cm high and 1.1 m wide at 160°C is exposed to ambient air at 20°C. Using the approximate solution, calculate the following:
   (i) Maximum velocity at 10 cm from the leading edge of the plate (ii) boundary layer thickness at 10 cm from the leading edge of plate (iii) local heat transfer coefficient at 10 cm from the leading edge of the plate (iv) average heat transfer coefficient over the surface of the plate (v) total mass flow through the boundary (vi) total heat loss from the plate, and (vii) temperature rise of air.
2. A hot plate 25 cm height and 100 cm width is exposed to atm. air at 20°C. The surface temperature of plate is 100°C. Find the heat loss from both the surfaces of the plate. If the height of the plate is changed to 50 cm, what will be the change in heat loss? Following empirical relation may be used: $Nu = 0.59(Gr \cdot Pr)^{1/4}$
   Properties of air at average temperature are:
   $\rho = 1.06$ kg/m^3; $\nu = 18.97 \times 10^{-6}$ m^2/s ; $C_p = 1004$ J/kgK ; $k = 0.029$ W/mK [M.U.]
3. A hot plate 100 cm height and 25 cm width is exposed to atm. air at 25°C. The surface temperature of plate is 95°C. Find the heat loss from both the surfaces of the plate. If the height of the plate is reduced to 50 cm and the width is increased to 40 cm, what will be the change in heat loss? Following empirical relation may be used:
   $Nu = C.(Gr \,.\, Pr)^m$ where
   $C = 0.59$ and $m = 1/4$ for $(Gr \cdot Pr) < 10^9$, and
   $C = 0.1$ and $m = 1/3$ for $(Gr \cdot Pr) > 10^9$
   Properties of air at average temperature are:
   $\rho = 1.06$ kg/m^3; $\nu = 18.97 \times 10^{-6}$ m^2/s ; $C_p = 1004$ J/kgK; $k = 0.029$ W/mK [M.U.]
4. A hot square plate 40 cm x 40 cm at 100°C is exposed to atmospheric air at 20°°C. Find the heat loses from both surfaces of the plate if:
   (a) the plate is held horizontal.
   (b) the plate is held in vertical plane.
   Properties of air at average temperature are: $\rho = 1.06$ kg/m^3; $\nu = 18.97 \times 10^{-6}$ m^2/s
   $C_p = 1004$ J/kgK; $k = 2.89 \times 10^{-2}$ W/mK;
   Following empirical relations may be used to find average heat transfer coefficients:
   Case(a): $Nu = 0.13(Gr \cdot Pr)^{1/4}$
   Case(b): For lower surface $Nu = 0.35(Gr \cdot Pr)^{1/4}$
   For upper surface $Nu = 0.71(Gr \cdot Pr)^{1/4}$ [M.U.]
5. A flat, vertical electrical heater is 0.5 m × 0.5 m in size and dissipates heat to still, ambient air at 20°C. Heat generation rate is 1 kW/m^2. Determine the average heat transfer coefficient and the average surface temperature.
6. A vertical steel plate, 0.5 m × 0.5 m in size and 3 mm thick, at an uniform temperature of 160°C, is exposed to atmospheric air at 20°C. Find the approximate time required for the plate to cool to 30°C, if the heat transfer coefficient in natural convection for the vertical plate is given by: $h = 1.42 \times (\Delta T/L)^{1/4}$. For steel, $\rho = 7800$ kg/m^3, $C_p = 473$ J/(kgK).
7. A 4 cm diameter steel ball at 160°C loses heat only by free convection to ambient air at 20°C. Calculate the time required for the temperature of the ball to reach 30°C. For steel, $\rho = 7800$ kg/m^3, $C_p = 473$ J/(kgK).
8. (a) A vertical pipe, 7.5 cm OD, 1.8 m long, has a surface temperature of 90°C. If the surrounding air is at 30°C, what is the rate of heat loss by free convection from this cylinder?
   (b) If the pipe is inclined to the vertical at an angle of 30 deg. during installation, how does the heat loss/m change?
9. A horizontal metal plate, 0.6 m x 0.6 m, is exposed to sun and receives radiant energy at the rate 170 W/m^2. If the heat transfer from the plate occurs to the surrounding air at 20°C by free convection only, find the steady state temperature of the plate. Assume that the bottom of the plate is insulated.
10. A horizontal, steam pipe of 10 cm OD runs through a room where the ambient air is at 20°C. If the outside surface of the pipe is at 160°C, and the emissivity of the surface is 0.85, find out the total heat loss per metre length of pipe.
11. A horizontal pipe carrying steam passes through a large room and is exposed to air at 30°C. The outer diameter of pipe is 20 cm. If the surface temperature of pipe is 200°C, find the loss of heat per metre length of the pipe by

convection and radiation. Assume emissivity of the pipe surface as 0.8. Properties of air given below:
$k = 0.0331$ W/mK; $\rho = 0.8826$ kg/m^3; $\nu = 24.83 \times 10^{-6}$; $C_p = 1014$ J/kgK.
Approximate empirical relation can be assumed as $Nu = 0.53\,(Ra)^{0.25}$ for $(10^3 < Ra < 10^9)$. [M.U.]

12. A tank contains water at 20°C. The water is heated by passing steam through a pipe placed in water. The pipe is 3 m long and 5 cm in diameter and its surface is maintained at 100°C. Find the heat loss from the pipe if the pipe is kept horizontal.
Following empirical relations may be used: $Nu = C.(Gr.Pr)^m$, where
$C = 0.53$ and $m = 0.25$ when $10^4 < Gr.Pr < 10^9$, and
$C = 0.13$ and $m = 1/3$ when $Gr.Pr > 10^9$.
13. A fine wire of 0.5 mm diameter is maintained at a constant temperature of 260°C by an electric current. The wire is exposed to air at 1 bar and 20°C. Calculate the heat transfer coefficient and the current flowing through the wire to maintain the wire temperature if the length of wire is 1 m. Electrical resistance of the wire is 8 ohms/m.
14. A spherical bulb of 5 cm diameter, with its surface temperature at 120°C, is exposed to still air at a temperature of 20°C. Determine the rate of convective heat loss.
15. A metal block is of 6 cm x 9 cm section and is 15 cm in height. Surface temperature of the block is 80°C. If it is exposed to air at 20°C, determine the rate of convective heat loss.
16. A circular disk heater of 3 cm diameter is maintained at a temperature of 60°C and is exposed to ambient air at 20°C. Calculate the free convection heat loss.
17. A short, solid vertical cylinder, 15 cm diameter and 15 cm high, is at 260°C and is exposed to air at 40°C. Estimate the value of average surface coefficient of heat transfer for the entire outside surface.
18. Solve Problem 10.11 using simplified equations for air. Refer to Table 10.3 for appropriate equation.
19. Helium gas at 2 bar pressure is contained between two horizontal panels separated by a distance of 20 mm. The lower panel is at a temperature of 80°C and the upper panel is at 20°C. Calculate the heat transfer rate by free convection per sq. m. of the panel surface.
20. Air gap between the two glass panels of a double-pane window (0.8 m wide × 1.5 m high) is 2 cms. If the two glass surfaces are at 25°C and – 5°C,
    (a) determine the rate of heat transfer through the window.
    (b) Verify the result with formula from Russian literature.
    (c) Show graphically how the heat transfer coefficient varies as the gap spacing.
21. Two vertical plates of size 40 cm × 40 cm are separated by a space of 3 cm and the gap is filled with water. Plate temperatures are 60°C and 20°C. What is the heat transfer rate? Verify the result with formula from Russian literature.
22. In a solar flat plate collector, the plate is 1.5 m high and 2.5 m wide, and is at a temperature of 120°C. The glass cover plate is at a distance of 3 cm from the collector surface and its temperature is 40°C. Space in between contains air at 1 atm. If the collector plate is inclined to the horizontal at 30 deg., determine the heat transfer coefficient and the rate of heat loss.
23. (a) Air at 1 bar fills the gap between two concentric spheres of 10 cm and 8 cm, respectively. Inner sphere is at 90°C and outer sphere is at 30°C. Calculate the free convection heat transfer across the gap. (b) Verify the result with formula from Russian literature.
24. A long tube of 0.2 m OD is maintained at 130°C. It is surrounded by a cylindrical radiation shield, located concentrically, such that the air gap between the two cylinders is 20 mm. The shield is at a temperature of 30°C. Estimate the convection heat transfer rate per metre length. Verify the result with formula from Russian literature.
25. A turbine blade is cooled by free convection with water as coolant. The cooling passage is 8 mm in diameter and 7 cm long. The blade velocity at a mean radius of 20 cm is 210 m/s. The hole surface temperature is at 200°C and cooling water temperature is 60°C. Find the average heat transfer coefficient and the rate of heat loss.
26. A 15 cm diameter steel shaft whose surface is at 150°C is allowed to cool while rotating about its own horizontal axis at 3 r.p.m. in an environment of air at 30°C. Find the initial rate of heat loss.
27. A 2 m diameter disk rotates at 600 r.p.m. has its surface at 60°C. Surrounding air is at 20°C. Find the value of convection coefficient and the rate of heat transfer from one side.
28. A sphere, 0.1 m in diameter is rotating at 30 r.p.m. in ambient air at 20°C. The sphere is at 160°C. Estimate the rate of heat transfer.
29. Consider a vertical heat sink with fins as shown in Fig. Problem 29.
The vertical heat sink, 0.35 m wide x 0.15 m high, is provided with vertical, rectangular fins of 1 mm thickness and 20 mm length. Base and surface temperature of fins is 80°C and the surrounding air is at 20°C. Determine the optimum fin spacing and the rate of heat transfer from the heat sink by natural convection.

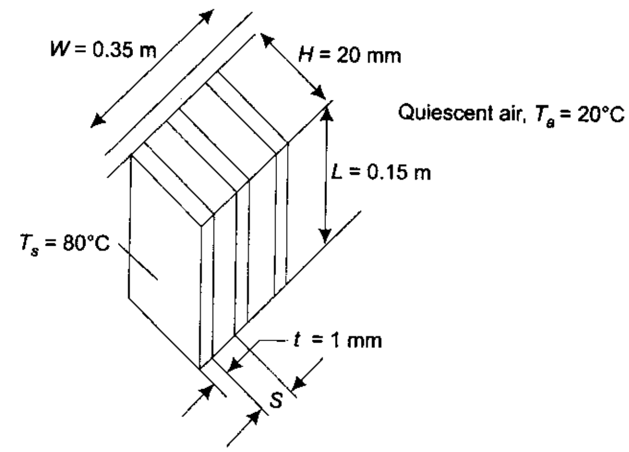

FIGURE Problem 10.29 Free convection from vertical heat sink with fins

30. Water at 20°C with a velocity of 5 cm/s flows across a horizontal cylinder maintained at a temperature of 60°C. Is the heat transfer by free convection significant? If so, calculate the rate of heat loss by combined free and forced convection. What will be the situation if the fluid is air at atmospheric pressure?
31. Consider a 3 m long vertical plate at a temperature of 80°C, kept in still air at 20°C. What is the forced motion velocity above which free convection heat transfer from the plate is negligible?
32. Atmospheric air flows through a 25 mm diameter horizontal tube at an average velocity of 25 cm/s. The tube is maintained at 150°C and the bulk air temperature is 30°C. Estimate the heat transfer coefficient if the tube is 0.35 m long.